

# Package ‘GoFKernel’

January 20, 2025

**Type** Package

**Title** Testing Goodness-of-Fit with the Kernel Density Estimator

**Version** 2.1-3

**Description** Tests of goodness-of-fit based on a kernel smoothing of the data.

References:

Pavía (2015) <[doi:10.18637/jss.v066.c01](https://doi.org/10.18637/jss.v066.c01)>.

**Encoding** UTF-8

**Depends** stats, KernSmooth

**License** GPL

**NeedsCompilation** no

**Author** Jose M. Pavía [aut, cre] (<<https://orcid.org/0000-0002-0129-726X>>)

**Maintainer** Jose M. Pavía <[jose.m.pavia@uv.es](mailto:jose.m.pavia@uv.es)>

**Repository** CRAN

**Date/Publication** 2024-12-06 10:20:07 UTC

## Contents

GoFKernel-package . . . . .	2
area.between . . . . .	2
density.reflected . . . . .	4
dgeometric.test . . . . .	5
fan.test . . . . .	7
inverse . . . . .	9
random.function . . . . .	10
risk76.1929 . . . . .	11
support.facto . . . . .	12
<b>Index</b>	<b>13</b>

GoFKernel-package

*Testing Goodness-of-fit with the Kernel Density Estimator*

---

**Description**

Tests of goodness-of-fit based on kernel smoothing of the data.

**Details**

Package: GoFKernel  
Depends: R (>=2.17.3), stats, KernSmooth (>=2.23-8)  
Type: Package  
Version: 2.1-1  
Date: 2018-05-26  
License: GPL

The most important functions in GoFKernel are [dgeometric.test](#) and [fan.test](#).

**Author(s)**

Jose M. Pavia

Maintainer: Jose M. Pavia <Jose.M.Pavia@uv.es>

**References**

- Fan, Y (1994) "Testing the goodness-of-fit of a parametric density function by kernel method", *Econometric Theory*, 10, 316-356.
- Li, O. and Racine, J.F. (2007) "Nonparametric Econometrics", Princeton University Press, New Jersey.
- Pavia, JM (2015) "Testing Goodness-of-fit with the Kernel Density Estimator: GoFKernel", *Journal of Statistical Software, Code Snippets*, 66(1), 1–27.

---

area.between*Area between a Density Function and a Kernel Estimate*

---

**Description**

The function `area.between` is an (internal) function of the GoFKernel package that calculates the area, in a given interval, between a theoretical density function and an empirical kernel estimate. `area.between` is called by `dgeometric.test` of the GoFKernel package.

**Usage**

```
area.between(f, kernel.density, lower = -Inf, upper = Inf)
```

**Arguments**

`f` a density function.

`kernel.density` an empirical kernel estimate, an object of the class `density`.

`lower` lower limit of the support of `f`, default `-Inf`.

`upper` upper limit of the support of `f`, default `Inf`.

**Details**

`area.between` is called by `dgeometric.test` and numerically calculates the area between the density function of the null hypothesis and the kernel density estimate of either the observed sample or a simulated sample from `f`.

**Value**

A number corresponding to the numerical value of the area between a density function and a kernel estimate.

**Author(s)**

Jose M. Pavia

**See Also**

[density.reflected](#), [dgeometric.test](#), [inverse.random.function](#), [support.facto](#) and [density](#)

**Examples**

```
## Unbounded example
x <- rnorm(100)
dx <- density(x)
area.between(dnorm, dx)

## Bounded example
x <- rbeta(100, 1.3, 2)
dx <- density.reflected(x, lower=0, upper=1)
area.between(dunif, dx)
```

---

density.reflected      *Kernel Density Estimation with Reflection*

---

### Description

The function `density.reflected` computes kernel density estimates for univariate observations using reflection in the borders.

### Usage

```
## S3 method for class 'reflected'
density(x, lower = -Inf, upper = Inf, weights= NULL, ...)
```

### Arguments

<code>x</code>	a numeric vector of data from which the estimate is to be computed.
<code>lower</code>	the lower limit of the interval to which <code>x</code> is theoretically constrained, default <code>-Inf</code> .
<code>upper</code>	the upper limit of the interval to which <code>x</code> is theoretically constrained, default <code>Inf</code> .
<code>weights</code>	numeric vector of non-negative observation weights, hence of same length as <code>x</code> . The default <code>NULL</code> is equivalent to <code>weights = rep(1/length(x), length(x))</code> .
<code>...</code>	further density arguments.

### Details

`density.reflected` is called by `dgeometric.test` and computes the density kernel estimate of a univariate random sample `x` of a random variable defined in the interval  $(lower, upper)$  using the default options of `density` and reflection in the borders. This avoids the density kernel estimate being underestimated in the proximity of `lower` or `upper`. For unbounded variables, `density.reflected` generates the same output as `density` with its default options.

### Value

An object of the class `density` with borders correction, whose underlying structure is a list containing the following components.

<code>x</code>	the <code>n</code> coordinates of the points where the density is estimated.
<code>y</code>	the estimated density values. These will be non-negative.
<code>bw</code>	the bandwidth used.
<code>n</code>	the sample size after elimination of missing values.
<code>call</code>	the call which produced the result.
<code>data.name</code>	the deparsed name of the <code>x</code> argument.
<code>has.na</code>	logical, for compatibility (always <code>FALSE</code> ).

The print method reports [summary](#) values on the `x` and `y` components.

**Note**

The function is based on [density](#).

**Author(s)**

Jose M. Pavia

**References**

Becker, R. A., Chambers, J. M. and Wilks, A. R. (1988) "The New S Language." Wadsworth & Brooks/Cole (for S version).

Scott, D. W. (1992) "Multivariate Density Estimation. Theory, Practice and Visualization." New York: Wiley.

Sheather, S. J. and Jones M. C. (1991) "A reliable data-based bandwidth selection method for kernel density estimation." J. Roy. Statist. Soc. B, 683–690.

Silverman, B. W. (1986) "Density Estimation." London: Chapman and Hall.

Venables, W. N. and Ripley, B. D. (2002) "Modern Applied Statistics with S." New York: Springer.

**See Also**

[dgeometric.test](#) and [density](#)

**Examples**

```
set.seed(234)
x <- runif(2000)
dx <- density.reflected(x,0,1)

## Plot of the density estimate with and without reflection
par(mfcol=c(1,2))
plot(dx, xlim=c(-0.1,1.1), ylim=c(0,1.1))
abline(h=1, col="red")

plot(density(x), xlim=c(-0.1,1.1), ylim=c(0,1.1))
abline(h=1, col="blue")
```

---

dgeometric.test

*Geometric Goodness-of-fit Test*

---

**Description**

Implementation of the goodness-of-fit test based on assessing the size of the area between the null hypothesis density function and a kernel density estimate of a sample.

**Usage**

```
dgeometric.test(x, fun.den, par = NULL, lower = -Inf, upper = Inf, n.sim = 101,
                bw=NULL)
```

**Arguments**

<code>x</code>	a numeric vector of data values for which the null hypothesis is tested.
<code>fun.den</code>	an actual density distribution function, such as <code>dnorm</code> . Only continuous densities are valid.
<code>par</code>	list of (additional) parameters of the density function under the null hypothesis, default <code>NULL</code> .
<code>lower</code>	lower end point of the support of the random variable defined by <code>fun.den</code> , default <code>-Inf</code> .
<code>upper</code>	upper end point of the support of the random variable defined by <code>fun.den</code> , default <code>-Inf</code> .
<code>n.sim</code>	number of iterations performed to calculate the <code>p.value</code> of the test, default 101.
<code>bw</code>	a number indicating the bandwidth to be used in the empirical kernel estimate of the data, default <code>NULL</code> . In its default option, the bandwidth varies in each simulated dataset and is the one estimated by default by <code>density</code> with a Gaussian kernel.

**Details**

`dgeometric.test` uses numerical integration and Monte Carlo simulation to implement the test based on assessing the extend of the area between a null hypothesis density function and a density kernel estimation. It works as follows. After computing by numerical integration the area between the density function under the null hypothesis and its sample empirical kernel estimate obtained using `density.reflected`, the `p-value` of the test is obtained by simulation as follows: (i) drawing `n.sim` samples from `fun.den` with the same size `length(x)` of our actual sample `x`; (ii) estimating the kernel density function for each of these new samples; (iii) computing the area between the theoretical density and each of the estimates obtained in (ii); and, (iv) calculating the `p-value` as the proportion of times the sample `n.sim` areas computed in (iii) exceed the value of the area computed from the observed sample.

**Value**

The output is an object of the class `htest` exactly like for the Kolmogorov-Smirnov test, `ks.test`. A list containing the following components:

<code>statistic</code>	the value of the test statistic.
<code>p.value</code>	the <code>p-value</code> of the test.
<code>method</code>	the character string "Geometric test".
<code>data.name</code>	a character string giving the name of the data.

**Note**

`dgeometric.test` calls `density.reflected` and `area.between` (and, in some circumstances, also `inverse`, `random.function` and `support.facto`), which are (internal) functions of the package `GoFKernel`.

**Author(s)**

Jose M. Pavia

**References**

Pavia, JM (2015) "Testing Goodness-of-fit with the Kernel Density Estimator: GoFKernel", Journal of Statistical Software, Code Snippets, 66(1), 1–27.

**See Also**

[area.between](#), [density.reflected](#), [inverse.random.function](#), [support.facto](#) and [fan.test](#).

**Examples**

```
set.seed(12)
x <- rlnorm(50, meanlog=1, sdlog=1)
## test if x follows a Gamma distribution with shape .6 and rate .1
dgeometric.test(x, dgamma, par=list(shape=0.6, rate=0.1), lower=0, upper=Inf, n.sim=100)

f0 <- function(x) ifelse(x>=0 & x<=1, 2-2*x, 0)
## test if risk76.1929 follows the distribution characterized by f0
dgeometric.test(risk76.1929, f0, lower=0, upper=1, n.sim=21)
```

---

fan.test

*Univariate implementation of the test of Fan (1994) in the form proposed by Li and Racine (2007).*

---

**Description**

Given a sample of a continuous univariate random variable and a density function `fun.den` with support in the interval  $(\text{lower}, \text{upper})$ , `fan.test` considers the test whose null hypothesis is that the sample has `fun.den` as density function and the test statistic and the corresponding p-value of the test based on the integral of the squared difference between the null hypothesis density function and a kernel smoothing approximation. To properly run, the `KernSmooth` package needs to be installed, as in its default option it depends on the `dpik` function to estimate the bandwidth.

**Usage**

```
fan.test(x, fun.den, par = NULL, lower = -Inf, upper = Inf, kernel = "normal",
        bw=NULL)
```

**Arguments**

`x` a numeric vector of data values for which the null hypothesis is tested.

`fun.den` an actual density distribution function, such as `dnorm`. Only continuous densities are valid.

par	list of (additional) parameters of the density function under the null hypothesis, default NULL.
lower	lower end point of the support of the random variable defined by fun.den, default -Inf.
upper	upper end point of the support of the random variable defined by fun.den, default -Inf.
kernel	a character string with the kernel to be used, either "normal" (a N(0,1) density), "box" (a uniform in -1 to 1) or "epanech" (a Epanechnikov quadratic kernel), default "normal".
bw	a number indicating the bandwidth to be used in the empirical kernel estimate of the data, default NULL. In its default option, the bandwidth is estimated using the dpik function included in the package KernSmooth.

### Details

The Fan's test is based on a normal approximation of the integral of the squared difference between the null hypothesis density function and a kernel smoothing approximation. In Li and Racine's form it is obtained as the aggregation of (i) a sampling component, (ii) the integrate of the square of the kernel convolution of the density null function and (iii) the sum of the convolution of the density in the sampled values, see Li and Racine (2007, pp.380-1) for details.

### Value

The output is an object of the class htest exactly like for the Kolmogorov-Smirnov test, [ks.test](#). A list containing the following components:

statistic	the value of the test statistic.
p.value	the p-value of the test.
method	the character string "Geometric test".
data.name	a character string giving the name of the data.

### Warning

fan.test calls the [dpik](#) function of KernSmooth

### Note

To properly run the function requires the package KernSmooth to be installed to estimate the bandwidth.

### Author(s)

Jose M. Pavia



## References

Fan, Y (1994) "Testing the goodness-of-fit of a parametric density function by kernel method", *Econometric Theory*, 10, 316–356.

Li, O. and Racine, J.F. (2007) "Nonparametric Econometrics", Princeton niversity Press, New Jersey.

## See Also

[dgeometric.test](#), [integrate](#) and [dpik](#).

## Examples

```
fan.test(runif(100), dunif, lower=0, upper=1)

f0 <- function(x) ifelse(x>=0 & x<=1, 2-2*x, 0)
## testing if risk76.1929 follows the distribution characterized by f0
fan.test(risk76.1929, f0, lower=0, upper=1, kernel="epanech")
```

---

inverse

*Inverse CDF Function*

---

## Description

Function to calculate the inverse function of a cumulative distribution function.

## Usage

```
inverse(f, lower = -Inf, upper = Inf)
```

## Arguments

f	a cdf function for which we want to obtain its inverse.
lower	the lower limit of f domain (support of the random variable), default -Inf.
upper	the upper limit of f domain (support of the random variable), default Inf.

## Details

`inverse` is called by `random.function` and calculates the inverse of a given function `f`. `inverse` has been specifically designed to compute the inverse of the cumulative distribution function of an absolutely continuous random variable, therefore it assumes there is only a root for each value in the interval  $(0,1)$  between  $f(\text{lower})$  and  $f(\text{upper})$ . It is for internal use in [dgeometric.test](#).

## Value

A function, the inverse function of a cumulative distribution function `f`.

**Note**

This function uses either `optim` with default options `method="L-BFGS-B"` or `uniroot` to derive the inverse function.

The upper endpoint must be strictly larger than the lower endpoint.

**Author(s)**

Jose M. Pavia

**References**

See the references in `optim` and `uniroot`.

**See Also**

`dgeometric.test`, `integrate`, `optim`, `random.function`, `support.facto` and `uniroot`.

**Examples**

```
f <- function(x) pbeta(x, shape1=2, shape2=3)
f.inv <- inverse(f, lower=0, upper=1)
f.inv(.2)
```

---

random.function

*Random Draw Generator*

---

**Description**

This function generates random draws of a continuous random variable given either its density or its cumulative distribution function.

**Usage**

```
random.function(n = 1, f, lower = -Inf, upper = Inf, kind = "density")
```

**Arguments**

n	number of draws, default 1.
f	either a density (default) or cumulative distribution function of the random variable.
lower	lower limit of the support of the random variable, default -Inf.
upper	upper limit of the support of the random variable, default Inf.
kind	character string with the function used to identify the distribution, either "density" (default) or "cumulative", as alternative.

**Details**

random.function uses the method of the inverse of the cdf to generate random draws from f.

**Value**

A vector of length n with n draws from a random variable with density (or cumulative distribution) function given by f.

**Note**

random.function is called by dgeometric.test when the corresponding r- function (random generator of f) is not available in the environment. random.function generates random samples from the null hypothesis density function specified in dgeometric.test.

**Author(s)**

Jose M. Pavia

**See Also**

[dgeometric.test](#), [integrate](#), [inverse](#) and [support.facto](#).

**Examples**

```
f0 <- function(x) ifelse(x>=0 & x<=1, 2-2*x, 0)
random.function(10, f0, lower=0, upper=1, kind="density")
```

---

risk76.1929

*Immigrants Exposed to Risk of Death*

---

**Description**

Vector containing the time exposed to risk of death with 76 years during 2006 for the 2006 registered Spanish immigrants born in 1929.

**Usage**

```
data(risk76.1929)
```

**Format**

The format is: num [1:362] 0.94 0.885 0.863 0.852 0.797 ...

**Note**

Under the null hypotheses of uniform distribution of date of birth and date of migration, this time exposed to risk is distributed as a  $f(x)=2-2x$   $0<x<1$ .

**Source**

Own elaboration from data available in [www.ine.es](http://www.ine.es)

**Examples**

```
plot(density.reflected(risk76.1929, 0, 1))
```

---

support.facto

*"De Facto" Support*

---

**Description**

support.facto computes the de facto numerical limits of a density function with theoretical infinite support. This function is an (internal) function of the GoFKernel package.

**Usage**

```
support.facto(f, lower = -Inf, upper = Inf)
```

**Arguments**

f	a density function.
lower	theoretical lower limit of the support of the random variable, default -Inf.
upper	theoretical upper limit of the support of the random variable, default, Inf.

**Details**

support.facto requires that the two first ordinary moments of f exist; otherwise, support.facto returns the introduced limits.

**Value**

A two components vector with the de facto lower and upper limits of f.

**Author(s)**

Jose M. Pavia

**See Also**

[area.between](#), [dgeometric.test](#), [inverse](#), [random.function](#) and [fan.test](#).

**Examples**

```
support.facto(dnorm)
```

# Index

- \* **datasets**
  - risk76.1929, 11
- \* **density**
  - area.between, 2
  - density.reflected, 4
- \* **htest**
  - dgeometric.test, 5
  - fan.test, 7
- \* **package**
  - GoFKernel-package, 2

area.between, 2, 6, 7, 12

density, 3, 5, 6

density (density.reflected), 4

density.reflected, 3, 4, 6, 7

dgeometric.test, 2, 3, 5, 5, 9–12

dpik, 8, 9

fan.test, 2, 7, 7, 12

GoFKernel (GoFKernel-package), 2

GoFKernel-package, 2

integrate, 9–11

inverse, 3, 6, 7, 9, 11, 12

ks.test, 6, 8

optim, 10

random.function, 3, 6, 7, 10, 10, 12

risk76.1929, 11

summary, 4

support.facto, 3, 6, 7, 10, 11, 12

uniroot, 10