Package ‘binomCI’

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Type Package

Title Confidence Intervals for a Binomial Proportion

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Depends R (>= 4.3.0)

Imports stats

Suggests Rfast, Rfast2

Description Twelve confidence intervals for one binomial proportion or a vector of binomial proportions are computed. The confidence intervals are: Jeffreys, Wald, Wald corrected, Wald, Blyth and Still, Agresti and Coull, Wilson, Score, Score corrected, Wald logit, Wald logit corrected, Arcsine and Exact binomial. References include, among others: Vollset, S. E. (1993). ``Confidence intervals for a binomial proportion''. Statistics in Medicine, 12(9): 809-824. <doi:10.1002/sim.4780120902>.

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Contents

binomCI-package *Confidence Intervals for a Binomial Proportion.*

Description

Functions to compute 12 confidence intervals for a binomial proportion.

Details

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Maintainers

Michail Tsagris <mtsagris@uoc.gr>.

Note

I would like to express my acknowledgements to Marc Girondot for spotting an error in the "Wilson" method in two extreme cases, when $x = 1$ and when $n - x = 1$. He also proposed a modification that exists in the package "Hmisc" and the relevant paper to cite is Agresti & Coull (1998).

Author(s)

Michail Tsagris <mtsagris@uoc.gr>.

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`binomCI` *Confidence Intervals for a Binomial Proportion.*

Description

Confidence Intervals for a Binomial Proportion.

Usage

```
binomCI(x, n, a = 0.05)
```

Arguments

`x`	The number of successes.
`n`	The number of trials.
`a`	The significance level to compute the $(1-\alpha)\%$ confidence intervals.

Details

The confidence intervals are:

Jeffreys:

$$\left[F(\alpha/2; x+0.5, n-x+0.5), F(1-\alpha/2; x+0.5, n-x+0.5)\right],$$

where $F(\alpha, a, b)$ denotes the α quantile of the Beta distribution with parameters a and b, $Be(a,b)$.

Wald:

$$\left[\hat{p} - Z_{1-\alpha/2} \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} - Z_{1-\alpha/2} \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right],$$

where $\hat{p} = \frac{x}{n}$ and $Z_{1-\alpha/2}$ denotes the $1-\alpha/2$ quantile of the standard normal distribution. If $\hat{p} = 0$ the interval becomes $(0, 1 - e^{\frac{1}{n}\log(\alpha 2)})$ and if $\hat{p} = 1$ the interval becomes $(e^{\frac{1}{n}\log(\alpha 2)}, 1)$.

Wald corrected:

$$\left[\hat{p} - Z_{1-\alpha/2} \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} - \frac{0.5}{n}, \hat{p} - Z_{1-\alpha/2} \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} + \frac{0.5}{n}\right],$$

and if $\hat{p} = 0$ or $\hat{p} = 1$ the previous (Wald) adjustment applies.

Wald BS:

$$\left[\hat{p} - Z_{1-\alpha/2} \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n - Z_{1-\alpha/2} - 2Z_{1-\alpha/2}/n - 1/n}} - \frac{0.5}{n}, \hat{p} - Z_{1-\alpha/2} \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n - Z_{1-\alpha/2} - 2Z_{1-\alpha/2}/n - 1/n}} + \frac{0.5}{n}\right],$$

and if $\hat{p} = 0$ or $\hat{p} = 1$ the previous (Wald) adjustment applies.

Agresti and Coull:

$$\left[\hat{\theta} - Z_{1-\alpha/2} \times \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n+4}}, \hat{p} - Z_{1-\alpha/2} \times \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n+4}}\right],$$

where $\hat{\theta} = \frac{x+2}{n+4}$.

Wilson:

$$\left[\frac{x_b}{n_b} - \frac{Z_{1-\alpha/2}\sqrt{n}}{n_b} \times \sqrt{\hat{p}(1-\hat{p}) + Z_{1-\alpha/2}/4}, \frac{x_b}{n_b} + \frac{Z_{1-\alpha/2}\sqrt{n}}{n_b} \times \sqrt{\hat{p}(1-\hat{p}) + Z_{1-\alpha/2}/4}\right],$$

where $x_b = x + Z^2_{1-\alpha/2}/2$ and $n_b = n + Z^2_{1-\alpha/2}$.

Score:

$$\left[\frac{x + Z^2_{1-\alpha/2} - c}{n + Z^2_{1-\alpha/2}}, \frac{x + Z^2_{1-\alpha/2} + c}{n + Z^2_{1-\alpha/2}}\right],$$

where $c = Z_{1-\alpha/2}\sqrt{x - x^2/n + Z^2_{1-\alpha/2}/4}$.

Score corrected:

$$\left[\frac{\ell_1}{n + Z_{1-\alpha/2}}, \frac{\ell_2}{n + Z_{1-\alpha/2}}\right],$$

where $\ell_1 = b_1 + 0.5Z^2_{1-\alpha/2} - Z_{1-\alpha/2}\sqrt{b_1 - b_1^2/n + 0.25Z^2_{1-\alpha/2}}$, $\ell_2 = b_2 + 0.5Z^2_{1-\alpha/2} + Z_{1-\alpha/2}\sqrt{b_2 - b_2^2/n + 0.25Z^2_{1-\alpha/2}}$ and $b_1 = x - 0.5$, $b_2 = x + 0.5$.

Wald-logit:

$$\left[1 - (1 + e^{b-c})^{-1}, 1 - (1 + e^{b+c})^{-1}\right],$$

where $b = \log(\frac{x}{n-x})$ and $c = \frac{Z_{1-\alpha/2}}{\sqrt{n\hat{p}(1-\hat{p})}}$. If $\hat{p} = 0$ or $\hat{p} = 1$ the previous (Wald) adjustment applies.

Wald-logit corrected:

$$\left[1-(1+e^{b-c})^{-1}, 1-(1+e^{b+c})^{-1}\right],$$

where $b=\log(\frac{\hat{p}_b}{\hat{q}_b})$, $\hat{p}_b=x+0.5$, $\hat{q}_b=n-x+0.5$ and $c=\frac{Z_{1-\alpha/2}}{\sqrt{(n+1)\frac{\hat{p}_b}{n+1}(1-\frac{\hat{p}_b}{n+1})}}$.

Arcsine:

$$\left\{\sin^2\left[sin^{-1}(\sqrt{\hat{p}})-0.5\frac{Z_{1-\alpha/2}}{\sqrt{n}}\right], \sin^2\left[sin^{-1}(\sqrt{\hat{p}})+0.5\frac{Z_{1-\alpha/2}}{\sqrt{n}}\right]\right\}.$$

If $\hat{p}=0$ or $\hat{p}=1$ the previous (Wald) adjustment applies.

Exact binomial:

$$\left[(1+\frac{a_1}{d_1})^{-1}, (1+\frac{a_2}{d_2})^{-1}\right],$$

where $a_1=n-x+1$, $a_2=a_1-1$, $d_1=x-F(\alpha/2, 2x, 2a_1)$, $d_2=(x+1)F(1-\alpha/2, 2(x+1), 2a_2)$ and $F(\alpha, a, b)$ denotes the α quantile of the F distribution with degrees of freedom a and b, $F(a, b)$.

Value

A list including:

`prop` The proportion.

`ci` A matrix with 12 rows containing the 12 different $(1-\alpha)\%$ confidence intervals.

Author(s)

Michail Tsagris.

R implementation and documentation: Michail Tsagris `<mtsagris@uoc.gr>`.

See Also

`binomCIs`

Examples

```
binomCI(45, 100)
```

`binomCIs` *Confidence Intervals for many Binomial Proportions.*

Description

Confidence Intervals for many Binomial Proportions.

Usage

```
binomCIs(x, n, a = 0.05)
```

Arguments

x	A vector with the number of successes.
n	A vector with the number of trials.
a	The significance level to compute the $(1-\alpha)\%$ confidence intervals.

Value

A list with the the first element being the vector with the proportions and the rest 12 items contain the $(1-\alpha)\%$ confidence intervals.

Author(s)

Michail Tsagris.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr>.

See Also

binomCI

Examples

```
x <- sample(40, 10)
n <- rep(40, 10)
binomCIs(x, n)
```

Index