Predicting methane and biogas production with the biogas package

Sasha D. Hafner (sasha.hafner@bce.au.dk) and Charlotte Rennuit

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Theory for predicting biogas production based on substrate composition and degradability is well-developed. But the calculations are tedious and seldom done. The 'biogas' package includes a single function predBg() to make these predictions. This document describes usage of the predBg() function through examples. In a final section, the methods behind the calculations are presented.

1 Getting started

To use the 'biogas' package, it must be installed on your computer. Since the package is available on CRAN (The Comprehensive R Archive Network)¹, it can be installed with the following command.

```
install.packages("biogas")
```

To use any installed package, it must be loaded. Use the next command to load the 'biogas' package.

library(biogas)

2 Methane production from defined substrates

2.1 Cellulose control in BMP test

Cellulose is often included in biochemical methane potential (BMP) experiments as a positive control—a comparison between the measured and expected quantity of CH₄ produced is used to evaluate the accuracy of the measurements. How much CH₄ should we expect from cellulose? To predict this we only need the chemical formula of cellulose.

¹You can download the package manually and find more information on getting started with the package at https://cran.r-project.org/web/packages/biogas/index.html.

```
predBg(form = "C6H1005")
## [1] 413.7274
```

So if it were all converted to $\mathrm{CH_4}$, we should see about 414 mL of $\mathrm{CH_4}$ (dry, at 101.325 kPa and 0°C)² per g of cellulose. This quantity is sometimes referred to as the theoretical BMP.

Since form is the first argument, we can easily omit the form = tag and use positional matching—we'll do this in the remainder of the document.

```
predBg("C6H1005")
## [1] 413.7274
```

By default, the result is per 1 g of substrate, but any mass can be used using the ${\tt mass}$ argument³.

You can see all the arguments and some default values using the arg() function.

```
args(predBg)

## function (form = NULL, mass = 1, mol = NULL, fs = 0, fd = 1,

## mcomp = NULL, COD = NULL, conc.sub = NULL, pH = NULL, temp = NULL,

## mu = 0.1, shortform = NULL, value = "CH4")

## NULL
```

The only required argument is form, or an alternative called mcomp. All these arguments will be described below. In this section, we'll focus on form and mass only.

Continuing with our example, to predict $\mathrm{CH_4}$ from 5.75 g of cellulose, set mass = 5.75.

```
predBg("C6H1005", mass = 5.75)
## [1] 2378.933
```

In this case we should expect about 2400 mL of CH₄. For a pure degradable compound like microcrystaline cellulose, it is reasonable to expect that 100% of the added mass is degraded by the microbial community. In other cases (e.g., when the substrate is plant biomass), we should reduce the degradability using the fd argument (which has a default of unity). But even for a substrate that is completely consumed by the bacteria and archaea in an anaerobic reactor (like microcrystaline cellulose) not all of the substrate is converted to CH₄. This is

²All volumes returned by the function are standardised to dry conditions at 0°C and 101.325 kPa (1.0 atm). They can be converted to different conditions using stdVol().

³Alternatively, the moles or total chemical oxygen demand (COD) can be specified with the mol and COD arguments.

beacuse the microorganisms use some fraction of it to synthesize their own cell biomass. The fraction of substrate that is used for cell synthesis can be set with the fs argument (which has a default of zero). In our example, we might expect fs between 5% and 25%, so we might find it useful to compare results from the calls below.

```
predBg("C6H1005", mass = 5.75, fs = 0)

## [1] 2378.933

predBg("C6H1005", mass = 5.75, fs = 0.05)

## [1] 2259.986

predBg("C6H1005", mass = 5.75, fs = 0.20)

## [1] 1903.146
```

But the function is vectorized in almost all arguments, so the single call below is a better option.

```
predBg("C6H1005", mass = 5.75, fs = c(0, 0.05, 0.2))
## [1] 2378.933 2259.986 1903.146
```

We can conclude that $\mathrm{CH_{4}}$ production from our 5.75 g of cellulose should be at least 1900 mL.

2.2 Designing a methanogen activity assay

Any chemical formula—for a pure compound, a mixture, or an empirical biomass formula—can be used for the form argument. For example, in methanogen activity tests, media typically include acetate, formate, or propionate as substrates. Let's assume we are developing an assay and want to determine how much substrate to use. We can start by comparing theoretical CH₄ production from each substrate on mass and molar bases.

```
predBg(c("CH00H", "CH3C00H", "CH3CH2C00H"), mol = 1)
## [1] 5590.147 22360.588 39131.029
predBg(c("CH00H", "CH3C00H", "CH3CH2C00H"), mass = 1)
## [1] 121.4563 372.3538 528.2409
```

From these results we can see that CH₄ production increases with the molecular mass of the acid, on both a mass and mole basis⁴. For a particular assay,

 $^{^4}$ This trend is consistent with the relative degree of oxidation: the O content is the same

assume we plan on using 60 mL of medium containing formic acid:acetic acid at 1:1 (mole basis) in a 100 mL serum bottle, and need at least 5 mL of $\rm CH_4$ for accurate quantification. We can specify predict $\rm CH_4$ production from such a mixture using the notation for form shown below.

```
predBg("(CH00H)0.5 (CH3C00H)0.5", mass = 1)
## [1] 263.4923
predBg("(CH00H)0.5 (CH3C00H)0.5", mol = 1)
## [1] 13975.37
```

What concentration should we use? We should have total production much higher than our 5 mL limit, since only a fraction of substrate is consumed during an activity assay. If we assume we need a predicted maximum of at least 50 mL, the required substrate quantity using the following call.

```
50/predBg("(CH00H)0.5 (CH3C00H)0.5", mol = 1)
## [1] 0.003577723
```

Based on this result, we could plan on 3.6 mmol of our combined substrate. Let's double-check to make sure it would provide a maximum of 50 mL of CH₄.

```
predBg("(CHOOH)0.5 (CH3COOH)0.5", mol = 0.0036)
## [1] 50.31132
```

A quantity of 3.6 mmol in a 60 mL solution requires a concentration of exactly 60 mmol $\rm L^{-1}$, which is therefore the concentration we should select based on our results.

3 Output options

So far all our examples have used the default value of the value argument to specify the type of output that is returned: "CH4". If we set the value argument to "all", more details are provided. Let's illustrate this with the acetic acid/formic acid example from above.

for these three acids is 2 moles per mole acid while the quantity of C and H increases with size.

In the ouput we can see the substrate formula (form) and mass (mass, g), along with its molar mass (mol.mass, g mol⁻¹), and calculated oxygen demand (COD, g). The response variable that we are interested in (and the one we worked with above) is the predicted CH_4 volume vCH4, in mL. The remaining columns show hydrolytic water consumption (hydro, g of H_2O) (in this example, H_2O is produced, not consumed), the molar fraction of CH_4 in the reaction products ($CH_4 + CO_2$) (fCH4), and the masses of CH_4 and CO_2 produced (mCH4 and mCO2, g).

In contrast to the default output (value = "CH4"), which is a vector, here output is a data frame. This difference isn't really apparent unless we get multiple sets of results. To illustrate, let's modify the previous call by adding four fs values. This addition will result in multiple rows in the output, as well as some additional columns.

```
predBg("(CHOOH)0.5 (CH3COOH)0.5", mol = 0.0036, fs = c(0, 0.01, 0.05, 0.1),
         value = "all")
##
                        form
                                  mass mol.mass
                                                           COD
                                                                 fs
                                                                      fe fd
                                                  moles
## 1 (CHOOH)0.5 (CH3COOH)0.5 0.1909404
                                          53.039 0.0036 0.144 0.00 1.00
  2 (CHOOH) 0.5 (CH3COOH) 0.5 0.1909404
                                          53.039 0.0036 0.144 0.01 0.99
  3 (CHOOH) 0.5 (CH3COOH) 0.5 0.1909404
                                          53.039 0.0036 0.144 0.05 0.95
    (CHOOH) 0.5 (CH3COOH) 0.5 0.1909404
                                          53.039 0.0036 0.144 0.10 0.90
           hydro
                      fCH4
                                vCH4
                                           mCH4
                                                     mCO2
  1 -0.01621440 0.4166667 50.31132 0.03609450 0.1386315 0.000000000
  2 -0.01686298 0.4159664 49.80821 0.03573356 0.1376413 0.001018017
## 3 -0.01945728 0.4130435 47.79576 0.03428978 0.1336804 0.005090085
  4 -0.02270016 0.4090909 45.28019 0.03248505 0.1287292 0.010180170
##
           N.req
## 1 0.00000000
## 2 0.000126063
## 3 0.000630315
## 4 0.001260630
```

Since fs is given, substrate partitioning parameters fs and fe (as well as degradability fd). Substrate partitioning parameters fs and fe are the fraction of substrate used for cell synthesis and energy production, and always sum to unity. With these parameters, biomass production (m.bio, g) and the reduced N requirement (NH₄⁺) for biomass production (N.req, g) can be calculate and are returned.

Now we have enough information to answer a second question: how much N is required in our medium? To get the most useful anser, let's switch our call a bit to use concentrations and not total masses. The only needed change is in mol, which we will switch from the total substrate quantity to the concentration (60 mmol $\rm L^{-1}$).

```
predBg("(CHOOH)0.5 (CH3COOH)0.5", mol = 0.06, fs = c(0, 0.05, 0.08, 0.1, 0.15),
         value = "all")
##
                                 mass mol.mass moles COD
                        form
                                                            fs
                                                                 fe fd
## 1 (CHOOH) 0.5 (CH3COOH) 0.5 3.18234
                                                0.06 2.4 0.00 1.00
                                        53.039
## 2 (CHOOH)0.5 (CH3COOH)0.5 3.18234
                                                0.06 2.4 0.05 0.95
                                        53.039
## 3 (CHOOH) 0.5 (CH3COOH) 0.5 3.18234
                                        53.039
                                                0.06 2.4 0.08 0.92
## 4 (CHOOH) 0.5 (CH3COOH) 0.5 3.18234
                                        53.039
                                                0.06 2.4 0.10 0.90
                                                                     1
  5 (CHOOH) 0.5 (CH3COOH) 0.5 3.18234
                                        53.039
                                                0.06 2.4 0.15 0.85
##
                                                  mCO2
          hydro
                     fCH4
                              vCH4
                                         mCH4
                                                            m.bio
                                                                        N.req
  1 -0.2702400 0.4166667 838.5220 0.6015750 2.310525 0.00000000 0.00000000
## 2 -0.3242880 0.4130435 796.5959 0.5714963 2.228006 0.08483475 0.01050525
## 3 -0.3567168 0.4107143 771.4403 0.5534490 2.178495 0.13573560 0.01680840
## 4 -0.3783360 0.4090909 754.6698 0.5414175 2.145487 0.16966950 0.02101050
## 5 -0.4323840 0.4047619 712.7437 0.5113388 2.062969 0.25450425 0.03151575
```

For methanogens, fs should not exceed 0.08[4], but to provide a safety factor, we could use the N need from the case where fs = 0.15. So we can conclude that we need at least 31 mg L^{-1} of NH_4^+ in our medium.

4 Unknown composition: using COD

If the composition of a substrate is not known, it is still possible to predict CH₄ production based on a measured chemical oxygen demand (COD). For example, assume we have a waste stream with a COD of 2600 mg L⁻¹. What is the maximum volume of CH₄ that could be produced?

```
predBg(COD = 2.6)
## [1] 908.3989
```

We should expect no more than 908 mL CH₄, and probably much less, depending on the degradability of our substrate. For example, perhaps we have data indicating that the organic matter in the waste stream is 60% degradable, and we assume that fs is 0.10.

```
predBg(COD = 2.6, fd = 0.6, fs = 0.1)
## [1] 490.5354
```

5 Complex substrates

Most real substrates are more complex than a single chemical compound such as cellulose or acetic acid. For example, both animal manure and wastewater sludge

are mixtures of many types of compounds. Composition of biomass substrates are sometimes described based on macromolecular characterization, as in [3] The mcomp argument⁵ is designed for this approach. The approach is based on default empirical formulas for macromolecular groups, such as carbohydrates and proteins. The mcomp argument is given as a named numeric vector, where values are relative or absolute mass-based concentrations within the substrate. For example, for cattle manure sample 2 in [3], we could use reported relative concentrations: carbohydrate, 682 mg g⁻¹; protein 158 mg g⁻¹; lipids, 54 mg g⁻¹; organic acids, 31 mg g⁻¹; and lignin, 75 mg g⁻¹ (all g per g organic matter) as shown below.

The predBg() function uses a single representative composition for each group and the relative masses provided to calculate an overall empirical formula. The empirical formula of the substrate, calculated using the provided macromolecular composition, can be seen by setting value = "all".

```
predBg(mcomp = c(carbohydrate = 0.682, protein = 0.158, lipid = 0.054,
                   VFA = 0.031, lignin = 0.075),
         mass = 1, fd = 0.4, fs = 0.1, value = "all")
                                form mass mol.mass
                                                         moles
                                                                     COD fs
## 1 C22.7452H36.9966O14.2950N1.0000
                                        1 553.1894 0.001807699 1.393915 0.1
     fe fd
                 hydro
                             fCH4
                                      vCH4
                                                mCH4
                                                          mCO2
## 1 0.9 0.4 0.08038705 0.5332337 175.3243 0.1257817 0.3020592 0.03941749
##
           N.rea
## 1 -0.005247033
```

To see which formulas are used for each macromolecular group, use shortform = FALSE. The ouput now shows an alternative expression for the empirical formula.

 $^{^5{\}rm Short}$ for "macromolecular composition" or "mass composition".

```
## vCH4 mCH4 mCO2 m.bio N.req
## 1 175.3242 0.1257816 0.3020595 0.03941747 -0.005247033
```

This approach is only needed if an empirical formula is not available for a substrate. If the elemental composition of a substrate has been determined, an empirical chemical formula can be calculated, and this formula can be used with $\tt predBg()$. For example, we could take the empirical formula calculated above, $C_{29.2587}H_{46.984}O_{18.9464}N_1$, round it for simplicity to $C_{29.3}H_{47}O_{18.9}N$ and use it as the form argument.

```
predBg("C29.2H47018.9N", mass = 1, fd = 0.4, fs = 0.1, shortform = FALSE, value = "all")
##
                                                    COD
                                                                         hydro
               form mass mol.mass
                                        moles
                                                        fs
                                                            fe
                                                                 fd
                          714.475 0.001399629 1.377235 0.1 0.9 0.4 0.07352903
##
  1 C29.2H47O18.9N
##
          fCH4
                   vCH4
                             mCH4
                                        mCO2
                                                              N.rea
                                                 m.bio
## 1 0.5296651 173.2263 0.1242765 0.3027526 0.0389458 -0.003019109
```

This results are slightly different than the result from above, but only because we rounded our formula.

The mcomp argument can also be used for mixtures of pure chemicals. For example, crude glycerol is a waste product from biodiesel production, and is often added to anaerobic digesters. It isn't uncommon for glycerol to make up >65% of the waste. We'll mix it with manure here, assuming a glycerol:manure VS ratio of 0.25:1.

Alternatively, since we now have an approximate empirical formula for this manure, we could use the following approach, which makes it easier to keep track of masses of the components of our mixture.

```
predBg(mcomp = c(C29.2H47018.9N = 0.8, C3H803 = 0.2), mass = 1, fd = 0.4, fs = 0.1)
## [1] 169.1733
```

We can see that predicted ${\rm CH_4}$ production per g of (mixed) substrate increases substantially when crude glycerol is added. Any chemical formula can be

used in this way through the mcomp argument. But one limitation at the present is that a single degradability parameter fd applies to the complete mixture.

Both form and mcomp can be used to specify mixed substrates. There are two differences between them: mcomp can be used with both macromolecular groups (e.g., carbohydrate) and chemical formulas, and quantities are mass-based, while form can only accept chemical formulas, and quantities are molebased. An example may clarify these differences. Assume we are working with a mixture of waste paper (primarily cellulose) and waste vegetable oil at a 5:1 ratio (dry mass basis). The easiest approach here is to use mcomp⁶.

```
predBg(mcomp = c(C6H1005 = 5, C54H10007 = 1), mass = 1)
### Warning in predBg(mcomp = c(C6H1005 = 5, C54H10007 = 1), mass =
1): Sum of mcomp != 1.0 so dividing all elements by the sum for calculation of formula.
### [1] 508.1105
```

To use form in this case, we need to translate this mass ratio to a mole ratio, which requires an additional step (where errors could be introduced). In this case the calculation is shown below.

```
1/5*molMass("C6H1005")/molMass("C54H10007")

## [1] 0.03764832

predBg("(C6H1005)1 (C54H10007)0.037648")

## [1] 508.1053
```

6 Total biogas production and composition

Predicting total biogas production is complicated by the behavior of CO_2 after it is produced. Unlike CH_4 , which is only sparingly soluble in water, a significant fraction of CO_2 produced remains in solution, and therefore does not contribute to biogas volume or affect its composition. Let's continue with the paper and vegetable oil example from the last section⁷.

 $^{^6}$ To avoid the resulting warning, be sure that the sum of the mcomp vector is unity.

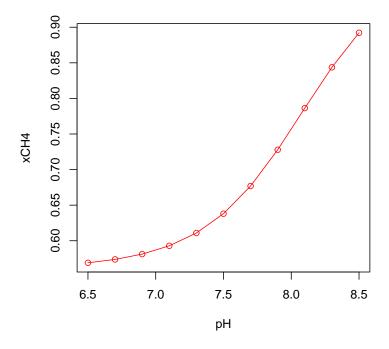
 $^{^7\}mathrm{But}$ change our masses to avoid a warning.

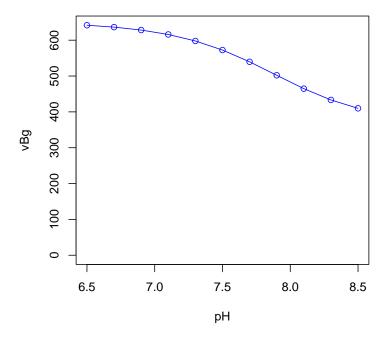
The fCH4 value is $\approx 55\%$ in our example, which is not the same as the CH₄ fraction in biogas. The difference is caused by a significant fraction of the CO₂ remaining in solution as CO₂ (aq), H₂CO₃, HCO₃⁻, and CO₃⁻². The predBg() function can also predict this partitioning of inorganic carbon between biogas and solution, if values are provided for three additional arguments: conc.sub, which is substrate concentration in g per kg H₂O; pH, which is the pH of the final solution; and temp, which is the system temperature in °C. Here, we'll assume the pH is well-buffered at pH 7.5, and our total substrate concentration is 50 g kg⁻¹. We will also assume that degradability is high (fd = 0.8) and substrate used for cell synthesis is typical (fs = 0.1).

```
predBg(mcomp = c(C6H1005 = 5/6, C54H10007 = 1/6), mass = 1,
         fd = 0.8, fs = 0.1, conc.sub = 50, pH = 7.5, temp = 35,
         value = "all")
                         form mass mol.mass
                                                 moles
                                                             COD fs
## 1 C1.52618H2.6151601.00000
                                 1 36.9655 0.02705225 1.454303 0.1 0.9 0.8
##
    conc.sub temp pH
                            hydro
                                       fCH4
                                                  xCH4
                                                           vCH4
                                                                    vC02
## 1
                35 7.5 0.09279292 0.5566161 0.6380063 365.8395 206.6652
          50
##
                   mCH4
                             mCO2
                                     mCO2Bg mCO2.sol
          vBg
                                                           cTIC
  1 572.5047 0.2624617 0.5735654 0.4085402 0.1650252 0.1874861 0.08225028
##
          N.req
## 1 0.01018521
```

Only after specifying values for these additional arguments will predBg return a column with biogas composition: xCH4 gives the mole fraction of CH4 in dry biogas. In this example, we can see that at 62% xCH4 is substantially higher than based on stoichiometry alone (fCH4). The difference between the two is, as explained above, TIC in solution. Additional new columns in this output give data for the volume of CO₂ in biogas (vCO2, mL), the total volume of (dry, standardized) biogas (vBg, mL), the mass of CO₂ in biogas and in solution (mCO2Bg and mCO2.sol, g), and the total inorganic carbon concentration in solution (cTIC, mol kg $^{-1}$). In fact, nearly one-third of all the CO₂ produced ends up in the solution in this example—it may be higher in other cases.

How might pH affect biogas composition? Let's look at a range of values.





Clearly, pH alone could have a major effect on biogas quality and volume! Predicting production and quality of biogas when the starting solution contains inorganic carbon is more complicated. The method assumes that all CO₂ in the system is accounted by the empirical formula (form). This may not be the case–for example if a bicarbonate (HCO₃) salt is included as a buffer in our BMP test with cellulose, or for our manure example, there is certainly significant TIC in raw manure. Here we need to be careful, and understand where our empirical formula came from. Does it include all C, or just the amount in the organic fraction? Does it include the O in TIC? Additional TIC could easily be added in with the predBg() function, but it is necessary to first determine the mole or mass ratio of substrate:TIC to correctly specify the resulting mixture using form or mcomp.

7 Calculation methods

How are the calculations presented above carried out? Stoichiometry of a complete biomethanation reaction is determined based on [4].

$$C_n H_a O_b N_c + (2n + c - b - \frac{9df_s}{20} - \frac{9f_e}{4}) H_2 O - >$$
 (1)

$$\frac{df_e}{8}CH_4 + (n - c - \frac{df_s}{5} - \frac{df_e}{8})CO_2 + \frac{df_s}{20}C_5H_7O_2N \tag{2}$$

$$+(c-\frac{df_s}{20})NH_4^+ + (c-\frac{df_s}{20})HCO_3^-$$
 (3)

Here, f_s and f_e are the fraction of substrate electrons going to cell synthesis and energy production, respectively, the formula $C_5H_7O_2N$ is an empirical formula for cell biomass⁸, and d = 4n + a - 2b - 3c. In addition to providing estimates of CH_4 production, this approach allows predBg() to return production of CO_2 , consumption or production of ammonia, and production of cell biomass. Predicted CH_4 is always expressed in mL (cm³) at standard conditions of 101.325 kPa (1.0 atm) and 0°C (273.15 K).

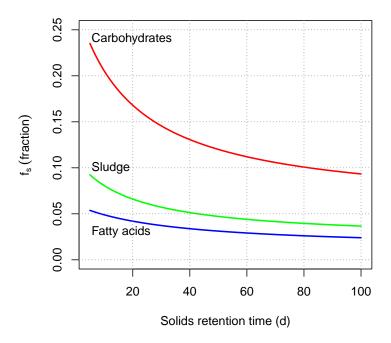
For these estimates to approach observed values, it is necessary to include and accurately set f_s . How should a f_s value be selected? Based on [4], f_s is related to the intrinsic value f_s^0 , the solids retention time θ_x , the rate of microbial biomass decay b (d⁻¹), and the degradability of microbial biomass f_{bd} (fraction).

$$f_s = f_s^0 \left(\frac{1 + (1 - f_{bd}) b\theta_x}{1 + b\theta_x} \right)$$
 (4)

Values of f_s^0 are given in [4]. For acetate fermenting methanogens, $f_s^0 = 0.05$, and for hydrogen oxidizing methanogens, $f_s^0 = 0.08$. But most calculations should be based on an overall value for the entire microbial community, which includes fermentative bacteria as well. Table 13.2 in [4] gives estimates for f_s^0 and b for various types of wastes: f_s^0 ranges from 0.06 for lipids to 0.28 for carbohydrates, and b ranges from 0.02 d⁻¹ for proteins to 0.05 for most other types of substrates. Calculated values of f_s are shown for three types of substrates below, assuming $f_{bd} = 0.8$ [4].

⁸Presently it is not possible to use a different formula.

 $^{^9 \}rm More$ accurately, total inorganic carbon (TIC), which includes both CO₂ and HCO $_3^-$ from the above reaction.



To determine "theororetical BMP", f_s should be set to zero ($f_e = 1$), which is the default.

Complex substrates are not completely degraded during anaerobic digestion, and the amount of the "degradable" (perhaps "degraded" is more accurate) fraction can be specified with the f_d argument. For complex substrates, the degradability of different components within a single substrate will differ, e.g., cellulose is much more degradable than lignin. But the current approach used for predBg() assumes that all components have the same degradability f_d .

The predBg() function can also calculate CH_4 production from COD. In this case, CH_4 volume is calculated based on the oxidation of CH_4 with $O_2^{10}[4]$:

$$1/8CH_4 + 1/4O_2 - > 1/8CO_2 + 1/4H_2O$$
 (5)

which gives a ratio of CH_4 to O_2 of 1/8:1/4=1:2. Based on conservation of COD, potential CH_4 production of a substrate with COD c is therefore:

$$V_{CH_4} = v_{CH_4}c/(M_{O_2}/2) \tag{6}$$

where $v_{CH_4} = \text{molar}$ volume of CH₄ (22361 mL g⁻¹, defined at 1 atm and 0°C [2]) and M_{O_2} is the molar mass of O₂ (32.0 g mol⁻¹). If f_e and f_d are available, they reduce the fraction of COD converted.

¹⁰This reaction is based on a single electron equivalent.

$$V_{CH_4} = f_d f_e v_{CH_4} c / (M_{O_2}/2) \tag{7}$$

The approach used to calculate CO_2 partitioning is based on equilibrium speciation as described in [1] and should be accurate for continuous reactors with stable operation at least. Predictions for batch reactors are more difficult and results will probably be less accurate.

References

- [1] S. D. Hafner, F. Montes, and C. A. Rotz. The role of carbon dioxide in emission of ammonia from manure. *Atmospheric Environment*, 66:63–71, 2012.
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