

# Package ‘elastes’

August 21, 2023

**Type** Package

**Title** Elastic Full Procrustes Means for Sparse and Irregular Planar Curves

**Version** 0.1.7

**Description** Provides functions for the computation of functional elastic shape means over sets of open planar curves. The package is particularly suitable for settings where these curves are only sparsely and irregularly observed. It uses a novel approach for elastic shape mean estimation, where planar curves are treated as complex functions and a full Procrustes mean is estimated from the corresponding smoothed Hermitian covariance surface. This is combined with the methods for elastic mean estimation proposed in Steyer, Stöcker, Greven (2022) <[doi:10.1111/biom.13706](https://doi.org/10.1111/biom.13706)>. See Stöcker et. al. (2022) <[arXiv:2203.10522](https://arxiv.org/abs/2203.10522)> for details.

**License** GPL (>= 3)

**Encoding** UTF-8

**Language** en-US

**Imports** elastics, utils, graphics, stats, splines, mgcv, sparseFLMM, orthogonalsplinebasis

**Suggests** knitr, covr, testthat (>= 3.0.0), rmarkdown, shapes

**RoxygenNote** 7.2.3

**Config/testthat/edition** 3

**VignetteBuilder** knitr

**URL** <https://mpff.github.io/elastes/>, <https://github.com/mpff/elastes>

**BugReports** <https://github.com/mpff/elastes/issues>

**NeedsCompilation** no

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**Repository** CRAN

**Date/Publication** 2023-08-21 15:00:29 UTC

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compute\_elastic\_shape\_mean

*Compute an elastic full Procrustes mean for a collection of curves*

---

**Description**

Computes an elastic full Procrustes mean for curves stored in `data_curves`. Constructor function for class `elastic_shape_mean`.

**Usage**

```
compute_elastic_shape_mean(
  data_curves,
  knots = seq(0, 1, len = 13),
  type = c("smooth", "polygon"),
  penalty = 2,
  var_type = c("smooth", "constant", "zero"),
  pfit_method = c("smooth", "polygon"),
  smooth_warp = function(i) 0,
  eps = 0.05,
  max_iter = 50,
  verbose = FALSE,
  cluster = NULL
)
```

**Arguments**

<code>data_curves</code>	list of data.frames with observed points in each row. Each variable is one coordinate direction. If there is a variable <code>t</code> , it is treated as the time parametrization, not as an additional coordinate.
<code>knots</code>	set of knots for the mean spline curve

type	if "smooth" linear srv-splines are used which results in a differentiable mean curve if "polygon" the mean will be piecewise linear.
penalty	the penalty to use in the covariance smoothing step. use '-1' for no penalty.
var_type	(experimental) assume "smooth", "constant" or "zero" measurement-error variance along t
pfit_method	(experimental) "smooth" or "polygon"
smooth_warp	(experimental) controls the weighting of original and smoothed observations over the iterations, if pfit_method == "smooth".
eps	the algorithm stops if L2 norm of coefficients changes by less than eps
max_iter	maximal number of iterations
verbose	print iterations
cluster	(experimental) use the parallel package for faster computation

### Value

an object of class `elastic_shape_mean`, which is a list with entries

type	"smooth" if mean was modeled using linear srv-splines, "polygon" if constant srv-splines
coefs	spline coefficients
knots	spline knots
variance	sample elastic shape variance
data_curves	list of <code>data.frames</code> with observed points in each row. First variable <code>t</code> gives the initial parametrization, second variable <code>t_optim</code> the optimal parametrization when the curve is aligned to the mean. Has the attributes 'rotation', 'scaling', 'translation' and 'dist_to_mean'. Use <a href="#">get_procrustes_fit</a> to get the elastic full Procrustes fit.
fit	see <code>fit_mean</code>

### Examples

```
curve <- function(t){
  rbind(t*cos(13*t), t*sin(13*t))
}
set.seed(18)
data_curves <- lapply(1:4, function(i){
  m <- sample(10:15, 1)
  delta <- abs(rnorm(m, mean = 1, sd = 0.05))
  t <- cumsum(delta)/sum(delta)
  data.frame(t(curve(t)) + 0.07*t*matrix(cumsum(rnorm(2*length(delta))),
    ncol = 2))
})

#randomly rotate and scale curves
rand_scale <- function(curve){ ( 0.5 + runif(1) ) * curve }
rand_rotate <- function(curve){
  names <- colnames(curve)
```

```

theta <- 2*pi*runif(1)
mat <- matrix(c(cos(theta), sin(theta), -sin(theta), cos(theta)), nrow = 2, ncol = 2)
curve.rot <- as.matrix(curve) %*% t(mat)
curve.rot <- as.data.frame(curve.rot)
colnames(curve.rot) <- names
return(curve.rot)
}
data_curves <- lapply(data_curves, rand_scale)
data_curves <- lapply(data_curves, rand_rotate)

#compute smooth procrustes mean with 2 order penalty
knots <- seq(0,1, length = 11)
elastic_shape_mean <- compute_elastic_shape_mean(
  data_curves,
  knots = knots,
  type = "smooth",
  penalty = 2
)
plot(elastic_shape_mean)

```

---

fit\_alignment\_proc2d *Optimal rotation and scaling alignment to a smooth curve*

---

## Description

Finds optimal rotation and scaling alignment for a discrete open srv curve to a smooth curve

## Usage

```

fit_alignment_proc2d(
  q,
  type,
  knots,
  var_type,
  coefs.compl,
  method,
  cov_fit,
  pca,
  L
)

```

## Arguments

q	complex srv curve with parametrization, needs to be vectorized. The result of a call to <code>get_model_data_complex</code>
type	spline degree
knots	basis knots
var_type	either "smooth" or "constant" measurement error in <code>cov_fit</code> object

coefs.compl	complex coefficients of smooth curve
method	temp
cov_fit	temp
pca	temp
L	temp

**Value**

optimal rotation G and scaling b

---

fit_mean	<i>Mean estimation for open planar curves.</i>
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---

**Description**

Fits an elastic full Procrustes mean for open, planar curves. Is usually called from [compute\\_elastic\\_shape\\_mean](#).

**Usage**

```
fit_mean(
  srv_data_curves,
  knots,
  penalty,
  var_type,
  pfit_method,
  max_iter,
  type,
  eps,
  cluster,
  verbose,
  smooth_warp
)
```

**Arguments**

srv_data_curves	list of data.frames with srv vectors in each row.curves
knots	set of knots for the mean spline curve
penalty	the penalty to use in the covariance smoothing step. use '-1' for no penalty.
var_type	(experimental) assume "smooth", "constant" or "zero" measurement-error variance along t
pfit_method	(experimental) "smooth" or "polygon"
max_iter	maximal number of iterations
type	if "smooth" linear srv-splines are used which results in a differentiable mean curve if "polygon" the mean will be piecewise linear.

eps	the algorithm stops if L2 norm of coefficients changes less
cluster	a cluster object for use in the bam call
verbose	print iterations
smooth_warp	(experimental) controls the weighting of original and smoothed observations over the iterations, if pfit_method == "smooth".

**Value**

a list with entries

type	"smooth" or "polygon"
coefs	coefs srv spline coefficients of the estimated mean
knots	spline knots
penalty	penalty used in the covariance estimation
distances	distances to mean
fit	a list containing t_optimoptimal parametrizations G_optimoptimal rotations b_optimoptimal scalings n_optimoptimal re-normalization n_iter number of iterations until convergence gram the mean basis Gram matrix, cov_fit the covariance smoothing objects in the final iteration, cov_pca cov coef matrix pca object in the final iteration and pfit_coefs the mean basis coefs of smoothed pfits in the final iteration

---

get_center	<i>Calculate the center of a curve</i>
------------	--

---

**Description**

Calculate the center of a curve

**Usage**

```
get_center(curve)
```

**Arguments**

curve	a data.frame with observed points in each row. Each variable is one coordinate direction. If there is a variable t, t_optim or id, it is treated as the time parametrization, not as an additional coordinate.
-------	--

**Value**

The average of observed points in curve.

---

get_distance	<i>Distance to a smooth curve</i>
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**Description**

Finds the distance of a discrete open srv curve to a smooth curve

**Usage**

```
get_distance(srv_curve, s, q, eps = 10 * .Machine$double.eps)
```

**Arguments**

srv_curve	srv transformation of the smooth curve, needs to be vectorized
s	time points for q, first has to be 0, last has to be 1
q	square root velocity vectors, one less than time points in s
eps	convergence tolerance

**Value**

distance between srv\_curve and q

---

get_evals	<i>Evaluate a curve on a grid</i>
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---

**Description**

Evaluate a curve on a grid

**Usage**

```
get_evals(curve, t_grid = NULL, ...)
```

```
## S3 method for class 'data.frame'  
get_evals(curve, t_grid = NULL, ...)
```

```
## S3 method for class 'elastic_shape_mean'  
get_evals(curve, t_grid = NULL, centering = TRUE, srv = FALSE, ...)
```

**Arguments**

curve	a one parameter function which is to be evaluated on a grid
t_grid	the curve is evaluated at the values in t_grid, first value needs to be 0, last value needs to be 1. If t_grid = NULL, a default regular grid with grid length 0.01 is chosen
...	other arguments
centering	TRUE if curves shall be centered
srv	TRUE if SRV curve shall be evaluated

**Value**

a data.frame with evaluations of the curve at the values in t\_grid in its rows.

**See Also**

See [get\\_evals](#) for the original code.

**Examples**

```
curve <- function(t){c(t*sin(10*t), t*cos(10*t))}
plot(get_evals(curve), type = "b")
```

---

get\_optimal\_t

*Finds optimal alignment for discrete open curves*

---

**Description**

Finds optimal aligned time points for srv curve q to srv curve p using coordinate wise optimization.

**Usage**

```
get_optimal_t(srv_procrustes_curves, coefs, t_optims, type, knots, eps, i)
```

**Arguments**

srv_procrustes_curves	scaling and rotation aligned srv curves
coefs	mean coefficients
t_optims	current optimal parametrization
type	"smooth" or "polygon"
knots	mean basis knots
eps	convergence tolerance
i	current iteration



**Value**

optimal time points for `srv_data_curves`, without first value 0 and last value 1 optimal time points have the distance of the observation to the `srv_curve` as an attribute

---

get_polygon_length	<i>Calculate the polygon length of a curve</i>
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**Description**

Calculate the polygon length of a curve

**Usage**

```
get_polygon_length(curve)
```

**Arguments**

curve	a <code>data.frame</code> with observed points in each row. Each variable is one coordinate direction. If there is a variable <code>t</code> , <code>t_optim</code> or <code>id</code> , it is treated as the time parametrization, not as an additional coordinate.
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**Value**

The length of curve, treating it as a polygon.

---

get_procrustes_fit	<i>Get Procrustes data curve from mean object.</i>
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**Description**

Compute the Procrustes aligned data curve...

**Usage**

```
get_procrustes_fit(data_curve)
```

**Arguments**

data_curve	A <code>data.frame</code> in an <code>elastic_shape_mean</code> object.
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**Value**

Aligned `data_curve` as a `data.frame`.

---

```
get_Procrustes_fit_from_param
```

*Helper functions for calculating Procrustes data curve from rotation, scaling and translation parameters.*

---

### Description

Compute the Procrustes fit given optimal rotation, scaling and translation.

### Usage

```
get_procrustes_fit_from_param(
  data_curve,
  rot,
  scale,
  plength,
  trans,
  norm_factor
)
```

### Arguments

data_curve	A data.frame with observed points on a curve. Each row is one point, each variable one coordinate direction. If there is a variable t, it is treated as the time parametrization, not as an additional coordinate.
rot	The rotation (in radian).
scale	The scaling.
plength	The polygon length of the original curve.
trans	The translation.
norm_factor	The normalization factor from the smooth curve estimate.

---

```
plot.elastic_shape_mean
```

*Plot method for planar elastic Procrustes mean curves*

---

### Description

Plots objects of class elastic\_shape\_mean.

### Usage

```
## S3 method for class 'elastic_shape_mean'
plot(x, srv = FALSE, centering = TRUE, asp = 1, col = "red", ...)
```

**Arguments**

<code>x</code>	object of class <code>elastic_shaped_mean</code> , usually a result of a call to <a href="#">compute_elastic_shape_mean</a>
<code>srv</code>	TRUE if the SRV curve should be plotted
<code>centering</code>	TRUE if mean and pfits should be centered
<code>asp</code>	numeric, giving the aspect ratio of the two coordinates, see <a href="#">plot.window</a> for details.
<code>col</code>	color of the mean curve.
<code>...</code>	further plotting parameters.

**Value**

No return value, called for side effects.

**See Also**

For examples see documentation of [compute\\_elastic\\_shape\\_mean](#). See [plot.elastic\\_mean](#) for the original code.

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