Package ‘rd2d’

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Type Package

Title Boundary Regression Discontinuity Designs

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URL https://rdpackages.github.io/rd2d/

Description
Provides estimation and inference procedures for boundary regression discontinuity (RD) designs using local polynomial methods, based on either bivariate coordinates or distance-based approaches.
Methods are developed in Cattaneo, Titiunik, and Yu (2025) <https://mdcattaneo.github.io/papers/Cattaneo-Titiunik-Yu_2025_BoundaryRD.pdf>.

Imports MASS, expm, ggplot2

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Maintainer Ruiqi Rae Yu <rae.yu@princeton.edu>

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Author Matias D. Cattaneo [aut],
Rocio Titiunik [aut],
Ruiqi Rae Yu [aut, cre]

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Contents

`rd2d-package` *rd2d: Two Dimensional Local Polynomial Regression Discontinuity Design*

Description

This package implements estimation and inference procedures for boundary regression discontinuity (RD) designs using local polynomial methods, based on either bivariate coordinates or distance-based approaches. Methods are developed in Cattaneo, Titiunik, and Yu (2025) `https://mdcattaneo.github.io/papers/Cattaneo-Titiunik-Yu_2025_BoundaryRD.pdf`.

Included functions are: rd2d for inference and estimation based on bivariate coordinates, rdbw2d for data-driven bandwidth selection based on bivariate coordinates, rd2d.dist for distance-based inference and estimation, rdbw2d.dist for distance-based bandwidth selection.

`print()` and `summary()` methods are available all four functions.

Related Stata, R, and Python packages useful for inference in RD designs are described in the following website:

https://rdpackages.github.io/

For an introduction to regression discontinuity design, see Cattaneo (2024) and references therein.

Author(s)

Matias Cattaneo, Princeton University. `<cattaneo@princeton.edu>`. Rocio Titiunik, Princeton University. `<titiunik@princeton.edu>`. Ruiqi Rae Yu, Princeton University. `<rae.yu@princeton.edu>`.

References

- Cattaneo, M. D., Titiunik, R., Yu, R. R. (2025). Estimation and Inference in Boundary Discontinuity Designs
- Cattaneo, M. D., Idrobo, N., Titiunik, R. (2024). A Practical Introduction to Regression Discontinuity Designs: Extensions

See Also

Useful links:

- `https://rdpackages.github.io/rd2d/`

print.rd2d — *Print Method for 2D Local Polynomial RD Estimation*

Description

Prints the results of a 2D local polynomial regression discontinuity (RD) estimation, as obtained from rd2d.

Usage

```
## S3 method for class 'rd2d'
print(x, ...)
```

Arguments

x	An object of class rd2d, returned by rd2d.
...	Additional arguments passed to the method (currently ignored).

Value

No return value. This function is called for its side effects, which are to print the rd2d results.

Author(s)

Matias D. Cattaneo, Princeton University. <cattaneo@princeton.edu>
Rocío Titiunik, Princeton University. <titiunik@princeton.edu>
Ruiqi Rae Yu, Princeton University. <rae.yu@princeton.edu>

See Also

rd2d for conducting 2D local polynomial RD estimation.

Supported methods: print.rd2d, summary.rd2d.

print.rd2d.dist — *Print Method for 2D Local Polynomial RD Estimation (Distance-Based)*

Description

Prints the results of a 2D local polynomial regression discontinuity (RD) estimation using distance-based evaluation, as obtained from rd2d.dist.

Usage

```
## S3 method for class 'rd2d.dist'
print(x, ...)
```

Arguments

`x`	An object of class `rd2d.dist`, returned by `rd2d.dist`.
`...`	Additional arguments passed to the method (currently ignored).

Value

No return value. This function is called for its side effects: it prints the `rd2d.dist` results.

Author(s)

Matias D. Cattaneo, Princeton University. <cattaneo@princeton.edu>
Rocío Titiunik, Princeton University. <titiunik@princeton.edu>
Ruiqi Rae Yu, Princeton University. <rae.yu@princeton.edu>

See Also

`rd2d.dist` for estimation using distance-based methods in 2D local polynomial RD designs.

Supported methods: `print.rd2d.dist`, `summary.rd2d.dist`.

`print.rdbw2d` *Print Method for Bandwidth Selection for 2D Local Polynomial RD Design*

Description

The print method for bandwidth selection for 2D local polynomial RD design

Usage

```
## S3 method for class 'rdbw2d'
print(x, ...)
```

Arguments

`x`	Class `rdbw2d` objects, obtained by calling `rdbw2d`.
`...`	Additional arguments passed to the method (currently ignored).

Value

No return value, called to print `rdbw2d` results.

Author(s)

Matias D. Cattaneo, Princeton University. <cattaneo@princeton.edu>
Rocío Titiunik, Princeton University. <titiunik@princeton.edu>
Ruiqi Rae Yu, Princeton University. <rae.yu@princeton.edu>

See Also

rdbw2d for bandwidth selection for 2D local polynomial RD design

Supported methods: print.rdbw2d, summary.rdbw2d.

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print.rdbw2d.dist *Print Method for Bandwidth Selection (Distance-Based) in 2D Local Polynomial RD Design*

Description

Print method for displaying summary information from distance-based bandwidth selection in 2D local polynomial regression discontinuity (RD) designs, as produced by rdbw2d.dist.

Usage

```
## S3 method for class 'rdbw2d.dist'
print(x, ...)
```

Arguments

x	An object of class rdbw2d.dist, returned by rdbw2d.dist.
...	Additional arguments passed to the method (currently ignored).

Value

No return value. This function is called for its side effects: it prints summary information of rdbw2d.dist.

Author(s)

Matias D. Cattaneo, Princeton University. <cattaneo@princeton.edu>
Rocío Titiunik, Princeton University. <titiunik@princeton.edu>
Ruiqi Rae Yu, Princeton University. <rae.yu@princeton.edu>

See Also

rdbw2d.dist for distance-based bandwidth selection in 2D local polynomial RD design.

Supported methods: print.rdbw2d.dist, summary.rdbw2d.dist.

rd2d *Two-Dimensional Local Polynomial Regression Discontinuity Design*

Description

rd2d implements bivariate local polynomial boundary regression discontinuity (RD) point estimators with robust bias-corrected pointwise confidence intervals and uniform confidence bands, developed in Cattaneo, Titiunik and Yu (2025). For robust bias-correction, see Calonico, Cattaneo and Titiunik (2014).

Companion commands are: rdbw2d for data-driven bandwidth selection.

For other packages of RD designs, visit https://rdpackages.github.io/

Usage

```
rd2d(
  Y,
  X,
  t,
  b,
  h = NULL,
  deriv = c(0, 0),
  tangvec = NULL,
  p = 1,
  q = 2,
 kernel = c("tri", "triangular", "epa", "epanechnikov", "uni", "uniform", "gau",
    "gaussian"),
  kernel_type = c("prod", "rad"),
  vce = c("hc1", "hc0", "hc2", "hc3"),
  masspoints = c("check", "adjust", "off"),
  C = NULL,
  level = 95,
  cbands = TRUE,
  side = c("two", "left", "right"),
  repp = 1000,
  bwselect = c("mserd", "imserd", "msetwo", "imsetwo", "user provided"),
  method = c("dpi", "rot"),
  bwcheck = 50 + p + 1,
  scaleregul = 3,
  scalebiascrct = 1,
  stdvars = TRUE
)
```

Arguments

Y Dependent variable; a numeric vector of length N, where N is the sample size.

`X`	Bivariate running variable (a.k.a score variable); a numeric matrix or data frame of dimension $N \times 2$, with each row $\mathbf{X}_i = (X_{1i}, X_{2i})$.
`t`	Treatment indicator; a logical or binary vector indicating treatment assignment ($t_i = 1$ if treated, $t_i = 0$ otherwise).
`b`	Evaluation points; a matrix or data frame specifying boundary points $\mathbf{b}_j = (b_{1j}, b_{2j})$, of dimension $J \times 2$.
`h`	Bandwidths. Either a positive scalar (same bandwidth for all dimensions and groups), or a matrix/data frame of size $J \times 4$, corresponding to $h_{\text{control},1}$, $h_{\text{control},2}$, $h_{\text{treated},1}$, $h_{\text{treated},2}$ at each evaluation point. If not specified, bandwidth `h` is computed by the companion command `rdbw2d`. Default is `h = NULL`.
`deriv`	The order of the derivatives of the regression functions to be estimated; a numeric vector of length 2 specifying the number of derivatives in each coordinate (e.g., $c(1, 2)$ corresponds to $\partial_1 \partial_2^2$).
`tangvec`	Tangent vectors; a matrix or data frame of dimension $J \times 2$ specifying directional derivatives. Overrides `deriv` if provided.
`p`	Polynomial order for point estimation ($p = 1$ by default).
`q`	Polynomial order for robust confidence interval construction. Must satisfy $q \geq p$; default is $q = p + 1$.
`kernel`	Kernel function to use. Options are `"unif"`, `"uniform"` (uniform), `"triag"`, `"triangular"` (triangular, default), and `"epan"`, `"epanechnikov"` (Epanechnikov).
`kernel_type`	Kernel structure. Either `"prod"` for product kernels or `"rad"` for radial kernels.
`vce`	Variance-covariance estimation method. Options are: • `"hc0"`: heteroskedasticity-robust plug-in residual variance estimator without small-sample adjustment. • `"hc1"`: heteroskedasticity-robust plug-in residual variance estimator with HC1 small-sample adjustment (default). • `"hc2"`: heteroskedasticity-robust plug-in residual variance estimator with HC2 adjustment. • `"hc3"`: heteroskedasticity-robust plug-in residual variance estimator with HC3 adjustment. Default is `"hc1"`.
`masspoints`	Handling of mass points in the running variable. Options are: • `"check"`: detects presence of mass points and reports the number of unique observations (default). • `"adjust"`: adjusts preliminary bandwidths to ensure a minimum number of unique observations within each side of the cutoff. • `"off"`: ignores presence of mass points.
`C`	Cluster ID variable used for cluster-robust variance estimation with degrees-of-freedom weights.Default is `C = NULL`.
`level`	Nominal confidence level for intervals/bands, between 0 and 100 (default is 95).
`cbands`	Logical. If `TRUE`, also compute uniform confidence bands (default is `FALSE`).

side Type of confidence interval. Options: "two" (two-sided, default), "left" (left tail), or "right" (right tail).

repp Number of repetitions for critical value simulation (used in uniform confidence bands). Default is 1000.

bwselect Bandwidth selection strategy. Options:

- "mserd". One common MSE-optimal bandwidth selector for the boundary RD treatment effect estimator for each evaluation point (default).
- "imserd". IMSE-optimal bandwidth selector for the boundary RD treatment effect estimator based on all evaluation points.
- "msetwo". Two different MSE-optimal bandwidth selectors (control and treatment) for the boundary RD treatment effect estimator for each evaluation point.
- "imsetwo". Two IMSE-optimal bandwidth selectors (control and treatment) for the boundary RD treatment effect estimator based on all evaluation points.
- "user provided". User-provided bandwidths. If h is not NULL, then bwselect is overwritten to "user provided".

method Bandwidth selection method for bias estimator based on local polynomials. Either "dpi" (default) for data-driven plug-in MSE optimal bandwidth selector or "rot" for rule-of-thumb bandwidth selector.

bwcheck If a positive integer is provided, the preliminary bandwidth used in the calculations is enlarged so that at least bwcheck unique observations are used. Default is 50 + p + 1.

scaleregul Scaling factor for the regularization term in bandwidth selection. Default is 3.

scalebiascrct Scaling factor used for bias correction based on higher order expansions. Default is 1.

stdvars Logical. If TRUE, the running variables X_{1i} and X_{2i} are standardized before computing the bandwidths. Default is FALSE. Standardization only affects automatic bandwidth selection if bandwidths are not manually provided via h.

Value

An object of class "rd2d", a list with components:

main A data frame with point estimates, variances, p-values, confidence intervals, confidence bands, and bandwidths at each evaluation point.

b1, b2 First and second coordinate of evaluation points $\mathbf{b} = (b_1, b_2)$.

Est.p Point estimate $\widehat{\tau}_p(\mathbf{b})$.

Var.p Variance of estimate $\widehat{\tau}_p(\mathbf{b})$.

Est.q Bias-corrected point estimate $\widehat{\tau}_q(\mathbf{b})$.

Var.q Variance of bias-corrected estimate $\widehat{\tau}_q(\mathbf{b})$.

p-value P-value based on t-statistic with bias-corrected estimate.

CI.lower, CI.upper Pointwise confidence intervals.

CB.lower, CB.upper Uniform confidence bands if computed.

- `h01`, `h02`, `h11`, `h12` Bandwidths used in each coordinate and group.
- `Nh0`, `Nh1` Effective sample size on each side of the cutoff.

`main.A0` Same structure as `main` but for control group outcomes.

`main.A1` Same structure as `main` but for treated group outcomes.

`tau.hat` Estimated treatment effect at each evaluation point.

`se.hat` Standard errors corresponding to estimates.

`cov.us` Covariance matrix used for uniform bands.

`cb` List with critical values, pointwise, and uniform intervals.

`pvalues` Two-sided p-values based on bias-corrected estimates.

`opt` List of options used in the function call.

Author(s)

Matias D. Cattaneo, Princeton University. <cattaneo@princeton.edu>
Rocío Titiunik, Princeton University. <titiunik@princeton.edu>
Ruiqi Rae Yu, Princeton University. <rae.yu@princeton.edu>

References

- Calonico, S., M. D. Cattaneo, and R. Titiunik. (2014). Robust Nonparametric Confidence Intervals for Regression-Discontinuity Designs
- Cattaneo, M. D., Titiunik, R., Yu, R. R. (2025). Estimation and Inference in Boundary Discontinuity Designs

See Also

`rdbw2d`, `print.rd2d`, `summary.rd2d`

Examples

```
# Simulated example
set.seed(123)
n <- 5000
X1 <- rnorm(n)
X2 <- rnorm(n)
t <- as.numeric(X1 > 0)
Y <- 3 + 2 * X1 + 1.5 * X2 + t + rnorm(n)
X <- cbind(X1, X2)
b <- matrix(c(0, 0, 0, 1), ncol = 2)

# Estimate treatment effect using rd2d
result <- rd2d(Y, X, t, b, cbands = TRUE)
print(result)
summary(result)
```

rd2d.dist *Local Polynomial RD Estimation on Distance-Based Running Variables*

Description

rd2d.dist implements distance-based local polynomial boundary regression discontinuity (RD) point estimators with robust bias-corrected pointwise confidence intervals and uniform confidence bands, developed in Cattaneo, Titiunik and Yu (2025). For robust bias-correction, see Calonico, Cattaneo and Titiunik (2014).

Companion commands are: rdbw2d.dist for data-driven bandwidth selection.

For other packages of RD designs, visit https://rdpackages.github.io/

Usage

```
rd2d.dist(
  Y,
  D,
  h = NULL,
  b = NULL,
  p = 1,
  q = 2,
  kink = c("off", "on"),
  kernel = c("tri", "triangular", "epa", "epanechnikov", "uni", "uniform", "gau",
    "gaussian"),
  level = 95,
  cbands = TRUE,
  side = c("two", "left", "right"),
  repp = 1000,
  bwselect = c("mserd", "imserd", "msetwo", "imsetwo", "user provided"),
  vce = c("hc1", "hc0", "hc2", "hc3"),
  rbc = c("on", "off"),
  bwcheck = 50 + p + 1,
  masspoints = c("check", "adjust", "off"),
  C = NULL,
  scaleregul = 1,
  cqt = 0.5
)
```

Arguments

Y Dependent variable; a numeric vector of length N, where N is the sample size.

D Distance-based scores $\mathbf{D}_i = (\mathbf{D}_i(\mathbf{b}_1), \cdots, \mathbf{D}_i(\mathbf{b}_J))$; dimension is $N \times J$ where N = sample size and J = number of cutoffs; non-negative values means data point in treatment group and negative values means data point in control group.

h
: Bandwidth(s); if $c = h$ then same bandwidth is used for both groups; if a matrix of size $J \times 2$ is provided, each row contains $(h_{\text{control}}, h_{\text{tr}})$ for the evaluation point; if not specified, bandwidths are selected via `rdbw2d.dist()`.

b
: Optional evaluation points; a matrix or data frame specifying boundary points $\mathbf{b}_j = (b_{1j}, b_{2j})$, dimension $J \times 2$.

p
: Polynomial order for point estimation. Default is `p = 1`.

q
: Polynomial order for bias-corrected estimation. Must satisfy $q \geq p$. Default is `q = p + 1`.

kink
: Logical; whether to apply kink adjustment. Options: `"on"` (default) or `"off"`.

kernel
: Kernel function to use. Options are `"unif"`, `"uniform"` (uniform), `"triag"`, `"triangular"` (triangular, default), and `"epan"`, `"epanechnikov"` (Epanechnikov).

level
: Nominal confidence level for intervals/bands, between 0 and 100 (default is 95).

cbands
: Logical. If `TRUE`, also compute uniform confidence bands (default is `FALSE`).

side
: Type of confidence interval. Options: `"two"` (two-sided, default), `"left"` (left tail), or `"right"` (right tail).

repp
: Number of bootstrap repetitions used for critical value simulation. Default is `1000`.

bwselect
: Bandwidth selection strategy. Options:

- `"mserd"`. One common MSE-optimal bandwidth selector for the boundary RD treatment effect estimator for each evaluation point (default).
- `"imserd"`. IMSE-optimal bandwidth selector for the boundary RD treatment effect estimator based on all evaluation points.
- `"msetwo"`. Two different MSE-optimal bandwidth selectors (control and treatment) for the boundary RD treatment effect estimator for each evaluation point.
- `"imsetwo"`. Two IMSE-optimal bandwidth selectors (control and treatment) for the boundary RD treatment effect estimator based on all evaluation points.
- `"user provided"`. User-provided bandwidths. If `h` is not `NULL`, then `bwselect` is overwritten to `"user provided"`.

vce
: Variance-covariance estimator for standard errors. Options:

`"hc0"` Heteroskedasticity-robust variance estimator without small sample adjustment (White robust).

`"hc1"` Heteroskedasticity-robust variance estimator with degrees-of-freedom correction. (default)

`"hc2"` Heteroskedasticity-robust variance estimator using leverage adjustments.

`"hc3"` More conservative heteroskedasticity-robust variance estimator (similar to jackknife correction).

rbc
: Logical. Whether to apply robust bias correction. Options: `"on"` (default) or `"off"`. When `kink = off`, turn on `rbc` means setting `q` to `p + 1`. When `kink = on`, turn on `rbc` means shrinking the bandwidth selector to be proportional to $N^{-1/3}$.

bwcheck — If a positive integer is provided, the preliminary bandwidth used in the calculations is enlarged so that at least bwcheck unique observations are used. Default is 50 + p + 1.

masspoints — Strategy for handling mass points in the running variable. Options:
- "check" (default) Check for repeated values and adjust inference if needed.
- "adjust" Adjust bandwidths to guarantee a sufficient number of unique support points.
- "off" Ignore mass points completely.

C — Cluster ID variable used for cluster-robust variance estimation with degrees-of-freedom weights.Default is C = NULL.

scaleregul — Scaling factor for the regularization term in bandwidth selection. Default is 1.

cqt — Constant controlling subsample fraction for initial bias estimation. Default is 0.5.

Details

MSE bandwidth selection for geometrical RD design

Value

An object of class "rd2d.dist", a list containing:

- main Data frame of point estimates, standard errors, confidence intervals, and bandwidths:
  - b1 First coordinate of the evaluation point.
  - b2 Second coordinate of the evaluation point.
  - Est.p Point estimate $\widehat{\tau}_{\text{dist},p}(\mathbf{b})$ with polynomial order p.
  - Var.p Variance of $\widehat{\tau}_{\text{dist},p}(\mathbf{b})$.
  - Est.q Bias-corrected estimate $\widehat{\tau}_{\text{dist},q}(\mathbf{b})$ with polynomial order q.
  - Var.q Variance of $\widehat{\tau}_{\text{dist},q}(\mathbf{b})$.
  - pvalue Two-sided p-value based on $T_{\text{dist},q}(\mathbf{b})$.
  - CI.lower Lower bound of confidence interval.
  - CI.upper Upper bound of confidence interval.
  - CB.lower Lower bound of uniform confidence band (if cbands=TRUE).
  - CB.upper Upper bound of uniform confidence band (if cbands=TRUE).
  - h0 Bandwidth used for control group ($D_i(\mathbf{b}) < 0$).
  - h1 Bandwidth used for treatment group ($D_i(\mathbf{b}) \geq 0$).
  - Nh0 Effective sample size for control group.
  - Nh1 Effective sample size for treatment group.
- main.A0 Summary table for the control group only.
- main.A1 Summary table for the treatment group only.
- tau.hat Vector of point estimates $\widehat{\tau}_p(\mathbf{b})$.
- se.hat Standard errors corresponding to $\widehat{\tau}_p(\mathbf{b})$.
- cb Confidence intervals and uniform bands.
- cov.us Covariance matrix used to construct uniform bands.
- opt A list of estimation options (e.g., p, q, kernel, level, etc.) and internal variables such as sample size N.

Author(s)

Matias D. Cattaneo, Princeton University. <cattaneo@princeton.edu>
Rocío Titiunik, Princeton University. <titiunik@princeton.edu>
Ruiqi Rae Yu, Princeton University. <rae.yu@princeton.edu>

References

- Cattaneo, M. D., Titiunik, R., Yu, R. R. (2025). Estimation and Inference in Boundary Discontinuity Designs

See Also

rdbw2d.dist, rd2d, print.rd2d.dist, summary.rd2d.dist

Examples

```
set.seed(123)
n <- 5000

# Generate running variables x1 and x2
x1 <- rnorm(n)
x2 <- rnorm(n)

# Define treatment assignment: treated if x1 >= 0
d <- as.numeric(x1 >= 0)

# Generate outcome variable y with some treatment effect
y <- 3 + 2 * x1 + 1.5 * x2 + 1.5 * d + rnorm(n, sd = 0.5)

# Define evaluation points (e.g., at the origin and another point)
eval <- data.frame(x.1 = c(0, 0), x.2 = c(0, 1))

# Compute Euclidean distances to evaluation points
dist.a <- sqrt((x1 - eval$x.1[1])^2 + (x2 - eval$x.2[1])^2)
dist.b <- sqrt((x1 - eval$x.1[2])^2 + (x2 - eval$x.2[2])^2)

# Combine distances into a matrix
D <- as.data.frame(cbind(dist.a, dist.b))

# Assign positive distances for treatment group, negative for control
d_expanded <- matrix(rep(2 * d - 1, times = ncol(D)), nrow = nrow(D), ncol = ncol(D))
D <- D * d_expanded

# Run the rd2d.dist function
result <- rd2d.dist(y, D, b = eval)

# View the estimation results
print(result)
summary(result)
```

rdbw2d *Bandwidth Selection for 2D Local Polynomial RD Design*

Description

rdbw2d implements bandwidth selector for bivariate local polynomial boundary regression discontinuity (RD) point estimators with robust bias-corrected pointwise confidence intervals and uniform confidence bands, developed in Cattaneo, Titiunik and Yu (2025).

Companion commands are: rd2d for point estimation and inference procedures.

For other packages of RD designs, visit https://rdpackages.github.io/

Usage

```
rdbw2d(
  Y,
  X,
  t,
  b,
  p = 1,
  deriv = c(0, 0),
  tangvec = NULL,
 kernel = c("tri", "triangular", "epa", "epanechnikov", "uni", "uniform", "gau",
    "gaussian"),
  kernel_type = c("prod", "rad"),
  bwselect = c("mserd", "imserd", "msetwo", "imsetwo"),
  method = c("dpi", "rot"),
  vce = c("hc1", "hc0", "hc2", "hc3"),
  bwcheck = 20,
  masspoints = c("check", "adjust", "off"),
  C = NULL,
  scaleregul = 1,
  scalebiascrct = 1,
  stdvars = FALSE
)
```

Arguments

Y Dependent variable; a numeric vector of length N, where N is the sample size.

X Bivariate running variable (a.k.a score variable); a numeric matrix or data frame of dimension $N \times 2$, with each row $\mathbf{X}_i = (X_{1i}, X_{2i})$.

t Treatment indicator; a logical or binary vector indicating treatment assignment ($t_i = 1$ if treated, $t_i = 0$ otherwise).

b Evaluation points; a matrix or data frame specifying boundary points $\mathbf{b}_j = (b_{1j}, b_{2j})$, of dimension $J \times 2$.

p Polynomial order of local polynomial estimator.

`deriv` The order of the derivatives of the regression functions to be estimated; a numeric vector of length 2 specifying the number of derivatives in each coordinate (e.g., $c(1, 2)$ corresponds to $\partial_1 \partial_2^2$).

`tangvec` Tangent vectors; a matrix or data frame of dimension $J \times 2$ specifying directional derivatives. Overrides `deriv` if provided.

`kernel` Kernel function to use. Options are `"unif"`, `"uniform"` (uniform), `"triag"`, `"triangular"` (triangular, default), and `"epan"`, `"epanechnikov"` (Epanechnikov).

`kernel_type` Kernel structure. Either `"prod"` for product kernels or `"rad"` for radial kernels.

`bwselect` Bandwidth selection strategy. Options:

- `"mserd"`. One common MSE-optimal bandwidth selector for the boundary RD treatment effect estimator for each evaluation point (default).
- `"imserd"`. IMSE-optimal bandwidth selector for the boundary RD treatment effect estimator based on all evaluation points.
- `"msetwo"`. Two different MSE-optimal bandwidth selectors (control and treatment) for the boundary RD treatment effect estimator for each evaluation point.
- `"imsetwo"`. Two IMSE-optimal bandwidth selectors (control and treatment) for the boundary RD treatment effect estimator based on all evaluation points.
- `"user provided"`. User-provided bandwidths. If `h` is not `NULL`, then `bwselect` is overwritten to `"user provided"`.

`method` Bandwidth selection method for bias estimator based on local polynomials. Either `"dpi"` (default) for data-driven plug-in MSE optimal bandwidth selector or `"rot"` for rule-of-thumb bandwidth selector.

`vce` Variance-covariance estimation method. Options are:

- `"hc0"`: heteroskedasticity-robust plug-in residual variance estimator without small-sample adjustment.
- `"hc1"`: heteroskedasticity-robust plug-in residual variance estimator with HC1 small-sample adjustment (default).
- `"hc2"`: heteroskedasticity-robust plug-in residual variance estimator with HC2 adjustment.
- `"hc3"`: heteroskedasticity-robust plug-in residual variance estimator with HC3 adjustment.

Default is `"hc1"`.

`bwcheck` If a positive integer is provided, the preliminary bandwidth used in the calculations is enlarged so that at least `bwcheck` unique observations are used. Default is `50 + p + 1`.

`masspoints` Handling of mass points in the running variable. Options are:

- `"check"`: detects presence of mass points and reports the number of unique observations (default).
- `"adjust"`: adjusts preliminary bandwidths to ensure a minimum number of unique observations within each side of the cutoff.
- `"off"`: ignores presence of mass points.

`C` Cluster ID variable used for cluster-robust variance estimation with degrees-of-freedom weights.Default is `C = NULL`.

`scaleregul` Scaling factor for the regularization term in bandwidth selection. Default is 3.

`scalebiascrct` Scaling factor used for bias correction based on higher order expansions. Default is 1.

`stdvars` Logical. If TRUE, the running variables X_{1i} and X_{2i} are standardized before computing the bandwidths. Default is `FALSE`. Standardization only affects automatic bandwidth selection if bandwidths are not manually provided via `h`.

Value

A list of class `"rdbw2d"` containing:

`bws` Data frame of estimated bandwidths for each evaluation point:

- `b1` First coordinate of the evaluation point.
- `b2` Second coordinate of the evaluation point.
- `h01` Estimated bandwidth for X_{1i} in the control group ($\mathcal{A}_0$).
- `h02` Estimated bandwidth for X_{2i} in the control group ($\mathcal{A}_0$).
- `h11` Estimated bandwidth for X_{1i} in the treatment group ($\mathcal{A}_1$).
- `h12` Estimated bandwidth for X_{2i} in the treatment group ($\mathcal{A}_1$).

`mseconsts` Data frame of intermediate quantities used in bandwidth calculation:

- `Nh0` Effective sample size for the control group $\mathcal{A}_0$.
- `Nh1` Effective sample size for the treatment group $\mathcal{A}_1$.
- `bias.0` Bias constant estimate for the control group.
- `bias.1` Bias constant estimate for the treatment group.
- `var.0` Variance constant estimate for the control group.
- `var.1` Variance constant estimate for the treatment group.
- `reg.bias.0` Bias correction adjustment for the control group.
- `reg.bias.1` Bias correction adjustment for the treatment group.
- `reg.var.0` Variance of the bias estimate for the control group.
- `reg.var.1` Variance of the bias estimate for the treatment group.

`opt` List containing:

- `p` Polynomial order used for estimation.
- `kernel` Kernel function used.
- `kernel_type` Type of kernel (product or radial).
- `stdvars` Logical indicating if standardization was applied.
- `bwselect` Bandwidth selection strategy used.
- `method` Bandwidth estimation method.
- `vce` Variance estimation method.
- `scaleregul` Scaling factor for regularization.
- `scalebiascrct` Scaling factor for bias correction.
- `N` Total sample size N.

Author(s)

Matias D. Cattaneo, Princeton University. <cattaneo@princeton.edu>
Rocío Titiunik, Princeton University. <titiunik@princeton.edu>
Ruiqi Rae Yu, Princeton University. <rae.yu@princeton.edu>

References

- Cattaneo, M. D., Titiunik, R., Yu, R. R. (2025). Estimation and Inference in Boundary Discontinuity Designs

See Also

rd2d, print.rdbw2d, summary.rdbw2d

Examples

```
# Simulated example
set.seed(123)
n <- 5000
X1 <- rnorm(n)
X2 <- rnorm(n)
t <- as.numeric(X1 > 0)
Y <- 3 + 2 * X1 + 1.5 * X2 + t + rnorm(n)
X <- cbind(X1, X2)
b <- matrix(c(0, 0, 0, 1), ncol = 2)

# MSE optimal bandwidth for rd2d
bws <- rdbw2d(Y, X, t, b)

# View the bandwidth selection results
print(bws)
summary(bws)
```

---

rdbw2d.dist — *Bandwidth Selection for Distance-Based RD Designs*

Description

rdbw2d.dist implements bandwidth selector for distance-based local polynomial boundary regression discontinuity (RD) point estimators with robust bias-corrected pointwise confidence intervals and uniform confidence bands, developed in Cattaneo, Titiunik and Yu (2025). For robust bias-correction, see Calonico, Cattaneo and Titiunik (2014).

Usage

```
rdbw2d.dist(
  Y,
  D,
```

```
  b = NULL,
  p = 1,
  kink = c("off", "on"),
  kernel = c("tri", "triangular", "epa", "epanechnikov", "uni", "uniform", "gau",
    "gaussian"),
  bwselect = c("mserd", "imserd", "msetwo", "imsetwo"),
  vce = c("hc1", "hc0", "hc2", "hc3"),
  bwcheck = 20 + p + 1,
  masspoints = c("check", "adjust", "off"),
  C = NULL,
  scaleregul = 1,
  cqt = 0.5
)
```

Arguments

`Y` Dependent variable; a numeric vector of length N, where N is the sample size.

`D` Distance-based scores $\mathbf{D}_i = (\mathbf{D}_i(\mathbf{b}_1), \cdots, \mathbf{D}_i(\mathbf{b}_J))$; dimension is $N \times J$ where N = sample size and J = number of cutoffs; non-negative values means data point in treatment group and negative values means data point in control group.

`b` Optional evaluation points; a matrix or data frame specifying boundary points $\mathbf{b}_j = (b_{1j}, b_{2j})$, dimension $J \times 2$.

`p` Polynomial order for point estimation. Default is `p = 1`.

`kink` Logical; whether to apply kink adjustment. Options: `"on"` (default) or `"off"`.

`kernel` Kernel function to use. Options are `"unif"`, `"uniform"` (uniform), `"triag"`, `"triangular"` (triangular, default), and `"epan"`, `"epanechnikov"` (Epanechnikov).

`bwselect` Bandwidth selection strategy. Options:

- `"mserd"`. One common MSE-optimal bandwidth selector for the boundary RD treatment effect estimator for each evaluation point (default).
- `"imserd"`. IMSE-optimal bandwidth selector for the boundary RD treatment effect estimator based on all evaluation points.
- `"msetwo"`. Two different MSE-optimal bandwidth selectors (control and treatment) for the boundary RD treatment effect estimator for each evaluation point.
- `"imsetwo"`. Two IMSE-optimal bandwidth selectors (control and treatment) for the boundary RD treatment effect estimator based on all evaluation points.
- `"user provided"`. User-provided bandwidths. If `h` is not `NULL`, then `bwselect` is overwritten to `"user provided"`.

`vce` Variance-covariance estimator for standard errors. Options:

`"hc0"` Heteroskedasticity-robust variance estimator without small sample adjustment (White robust).

`"hc1"` Heteroskedasticity-robust variance estimator with degrees-of-freedom correction. (default) "hc2" Heteroskedasticity-robust variance estimator using leverage adjustments.

"hc3" More conservative heteroskedasticity-robust variance estimator (similar to jackknife correction).

bwcheck If a positive integer is provided, the preliminary bandwidth used in the calculations is enlarged so that at least bwcheck unique observations are used. Default is 50 + p + 1.

masspoints Strategy for handling mass points in the running variable. Options:

"check" (default) Check for repeated values and adjust inference if needed.

"adjust" Adjust bandwidths to guarantee a sufficient number of unique support points.

"off" Ignore mass points completely.

C Cluster ID variable used for cluster-robust variance estimation with degrees-of-freedom weights.Default is C = NULL.

scaleregul Scaling factor for the regularization term in bandwidth selection. Default is 1.

cqt Constant controlling subsample fraction for initial bias estimation. Default is 0.5.

Value

An object of class "rdbw2d.dist", containing:

bws Data frame of optimal bandwidths for each evaluation point:

- b1 First coordinate of the evaluation point $b1$.
- b2 Second coordinate of the evaluation point $b2$.
- h0 Bandwidth for observations with distance $D_i(\mathbf{b}) < 0$.
- h1 Bandwidth for observations with distance $D_i(\mathbf{b}) \geq 0$.
- Nh0 Effective sample size for $D_i(\mathbf{b}) < 0$.
- Nh1 Effective sample size for $D_i(\mathbf{b}) \geq 0$.

mseconsts Data frame of intermediate bias and variance constants used for MSE/IMSE calculations.

opt A list of options and settings used in estimation, including p, kernel, sample size N, and user-specified choices.

Author(s)

Matias D. Cattaneo, Princeton University. <cattaneo@princeton.edu>
Rocío Titiunik, Princeton University. <titiunik@princeton.edu>
Ruiqi Rae Yu, Princeton University. <rae.yu@princeton.edu>

References

- Cattaneo, M. D., Titiunik, R., Yu, R. R. (2025). Estimation and Inference in Boundary Discontinuity Designs

See Also

rd2d.dist, rd2d, summary.rdbw2d.dist, print.rdbw2d.dist

Examples

```
set.seed(123)
n <- 5000

# Generate running variables x1 and x2
x1 <- rnorm(n)
x2 <- rnorm(n)

# Define treatment assignment: treated if x1 >= 0
d <- as.numeric(x1 >= 0)

# Generate outcome variable y with some treatment effect
y <- 3 + 2 * x1 + 1.5 * x2 + 1.5 * d + rnorm(n, sd = 0.5)

# Define evaluation points (e.g., at the origin and another point)
eval <- data.frame(x.1 = c(0, 0), x.2 = c(0, 1))

# Compute Euclidean distances to evaluation points
dist.a <- sqrt((x1 - eval$x.1[1])^2 + (x2 - eval$x.2[1])^2)
dist.b <- sqrt((x1 - eval$x.1[2])^2 + (x2 - eval$x.2[2])^2)

# Combine distances into a matrix
D <- as.data.frame(cbind(dist.a, dist.b))

# Assign positive distances for treatment group, negative for control
d_expanded <- matrix(rep(2 * d - 1, times = ncol(D)), nrow = nrow(D), ncol = ncol(D))
D <- D * d_expanded

# Run the rd2d.dist function
bws <- rdbw2d.dist(y, D, b = eval)

# View the estimation results
print(bws)
summary(bws)
```

---

summary.rd2d *Summary Method for 2D Local Polynomial RD Estimation*

Description

Summarizes estimation and bandwidth results from a 2D local polynomial regression discontinuity (RD) design, as produced by rd2d.

Usage

```
## S3 method for class 'rd2d'
summary(object, ...)
```

Arguments

`object`	An object of class `rd2d`, typically returned by `rd2d`.
`...`	Optional arguments. Supported options include:

- `CBuniform`: Logical. If `TRUE`, displays uniform confidence bands; if `FALSE` (default), displays pointwise confidence intervals.
- `subset`: Integer vector of indices of evaluation points to display. Defaults to all evaluation points.
- `output`: Character. Use `"main"` to display estimation results, or `"bw"` to display bandwidth information. Default is `"main"`.
- `sep`: Integer vector of length three. Controls spacing in the output. `sep[1]` controls spacing for the columns of bandwidths, estimation, z-value, and p-value in the `"main"` table. `sep[2]` controls spacing for the confidence interval (confidence bands) in the `"main"` table. `sep[3]` controls spacing for the columns in the `"bw"` table. Default is `c(7, 17, 8)`.

Value

No return value. This function is called for its side effects: it prints a formatted summary of `rd2d` results.

Author(s)

Matias D. Cattaneo, Princeton University. `<cattaneo@princeton.edu>`
Rocío Titiunik, Princeton University. `<titiunik@princeton.edu>`
Ruiqi Rae Yu, Princeton University. `<rae.yu@princeton.edu>`

See Also

`rd2d` for estimation using 2D local polynomial RD design.

Supported methods: `print.rd2d`, `summary.rd2d`.

`summary.rd2d.dist` *Summary Method for 2D Local Polynomial RD Estimation (Distance-Based)*

Description

Summarizes estimation and bandwidth results from a 2D local polynomial regression discontinuity (RD) design using distance-based methods, as returned by `rd2d.dist`.

Usage

```
## S3 method for class 'rd2d.dist'
summary(object, ...)
```

Arguments

object	An object of class rd2d.dist, returned by rd2d.dist.
...	Optional arguments. Supported options include:

- CBuniform: Logical. If TRUE, displays uniform confidence bands; if FALSE (default), displays pointwise confidence intervals.
- subset: Integer vector of indices of evaluation points to display. Defaults to all evaluation points.
- output: Character. Use "main" to display estimation results, or "bw" to display bandwidth information. Default is "main".
- sep: Integer vector of length three. Controls spacing in the output. sep[1] controls spacing for the columns of boundary points, estimation, z-value, and p-value in the "main" table. sep[2] controls spacing for confidence intervals (or bands) in the "main" table. sep[3] controls spacing for the columns in the "bw" table. Default is c(7, 17, 8).

Value

No return value. This function is called for its side effects: it prints a formatted summary of rd2d.dist results.

Author(s)

Matias D. Cattaneo, Princeton University. <cattaneo@princeton.edu>
Rocío Titiunik, Princeton University. <titiunik@princeton.edu>
Ruiqi Rae Yu, Princeton University. <rae.yu@princeton.edu>

See Also

rd2d.dist for estimation using distance-based 2D local polynomial RD design.

Supported methods: print.rd2d.dist, summary.rd2d.dist.

---

summary.rdbw2d — *Summary Method for Bandwidth Selection for 2D Local Polynomial RD Design*

Description

Summary method for objects of class rdbw2d, displaying bandwidth selection results for 2D local polynomial regression discontinuity designs.

Usage

```
## S3 method for class 'rdbw2d'
summary(object, ...)
```

Arguments

object
: An object of class rdbw2d, typically returned by rdbw2d.

...
: Optional arguments. Supported options include:
  - subset: Integer vector of indices of evaluation points to display. Defaults to all evaluation points.
  - sep: Integer. Controls spacing in the output. Default is 8.

Value

No return value. Called for its side effects of printing a formatted summary of rdbw2d results.

Author(s)

Matias D. Cattaneo, Princeton University. <cattaneo@princeton.edu>
Rocío Titiunik, Princeton University. <titiunik@princeton.edu>
Ruiqi Rae Yu, Princeton University. <rae.yu@princeton.edu>

See Also

rdbw2d for bandwidth selection in 2D local polynomial RD design.

Supported methods: print.rdbw2d, summary.rdbw2d.

---

summary.rdbw2d.dist — *Summary Method for Bandwidth Selection in 2D Local Polynomial RD Design (Distance-Based)*

Description

Summarizes bandwidth selection results from a 2D local polynomial regression discontinuity (RD) design using distance-based methods, as returned by rdbw2d.dist.

Usage

```
## S3 method for class 'rdbw2d.dist'
summary(object, ...)
```

Arguments

object
: An object of class rdbw2d.dist, returned by rdbw2d.dist.

...
: Optional arguments. Supported options include:
  - subset: Integer vector of indices of evaluation points to display. Defaults to all evaluation points.
  - sep: Integer vector of length two. Controls spacing in the output. sep[1] controls spacing for the columns of evaluation points in the table. sep[2] controls spacing for the columns of bandwidths in the table. Default is c(8, 14).

Value

No return value. This function is called for its side effects: it prints a formatted summary of `rdbw2d.dist` results.

Author(s)

Matias D. Cattaneo, Princeton University. <cattaneo@princeton.edu>
Rocío Titiunik, Princeton University. <titiunik@princeton.edu>
Ruiqi Rae Yu, Princeton University. <rae.yu@princeton.edu>

See Also

`rdbw2d.dist` for bandwidth selection using 2D local polynomial RD design with distance-based methods.

Supported methods: `print.rdbw2d.dist`, `summary.rdbw2d.dist`.

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